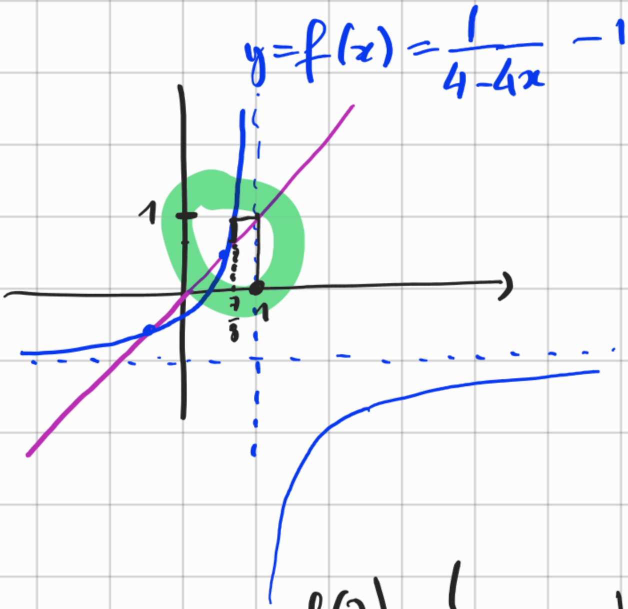
ANALISI MATEMATICA B
LEZIONE 34 - 10.12.2021

Es 5 test settimanale

$$\begin{cases} a_0 = \alpha \\ a_{n+1} = \dfrac{1}{4-4a_n} - 1 \end{cases}$$



$$\frac{1}{4-4x} - 1 = x$$

$$\frac{1}{4-4x} = x+1$$

$$1 = (x+1)(4-4x)$$

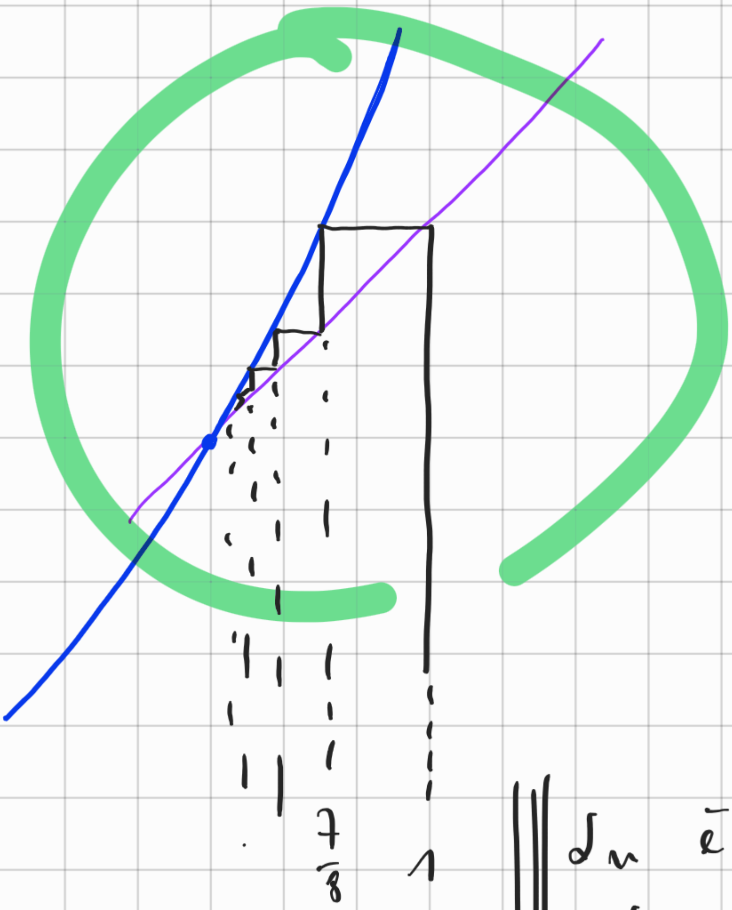
$$1 = \cancel{4x} - 4x^2 + 4 - \cancel{4x}$$

$$4x^2 = 3 \qquad x = \pm\frac{\sqrt{3}}{2}$$

$$f\left(\frac{7}{8}\right) = \frac{1}{4-\frac{7}{2}} - 1$$

$$= 2 - 1 = 1$$

$$\nexists\ f(1)$$



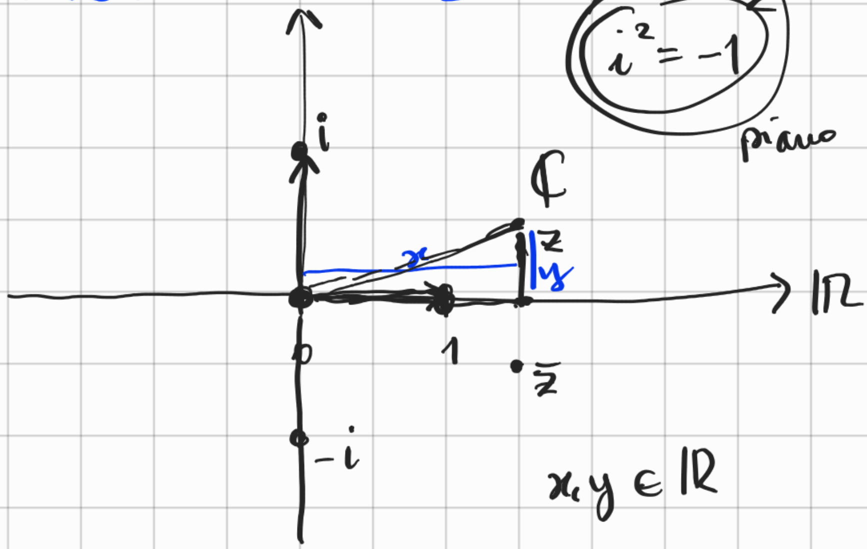
α_n

$$\begin{cases} \alpha_0 = 1 \\ \alpha_{n+1} = f^{-1}(\alpha_n) \end{cases}$$

α_n è decrescente

$\alpha_n \to \frac{\sqrt{3}}{2}$

NUMERI COMPLESSI



$$i^2 = -1$$

$$(-i)^2 = (-1)^2 i^2 = i^2 = -1$$

piano di Gauss

$$\mathbb{R} \subseteq \mathbb{C}$$

$$\underbrace{x \qquad x + i\cdot 0}$$

$x, y \in \mathbb{R}$

$$\mathbb{C} \cong \mathbb{R}\times\mathbb{R}$$

$$x+iy \qquad (x,y)$$

$z \in \mathbb{C}$ $\qquad z = x\cdot 1 + y\cdot i = \underbrace{x+iy}$

$z = x+iy \qquad w = a+ib \qquad z+w = x+iy+a+ib$

$x = \mathrm{Re}\, z \qquad y = \mathrm{Im}\, z \qquad = (x+a) + i(y+b)$

$$z\cdot w = (\underbrace{x+iy})\cdot(\underbrace{a+ib}) = x\cdot a + xib + iya + iyib$$

$$= \underbrace{xa - yb} + i\underbrace{(xb+ya)}$$

$0 = 0 + i\cdot 0$

$1 = 1 + i\cdot 0$

Si può dimostrare facilmente che $+$, $\cdot$ soddisfano tutte le proprietà di "anello".

- Ogni $z = x+iy$, $z \neq 0$, ha inverso: $\exists w : z\cdot w = 1$.

Coniugio $\qquad z = x+iy \qquad \bar{z} = z^* = x - iy$

$\overline{z+w} = \bar{z} + \bar{w}$ $\qquad$ ovvio

$\overline{z\cdot w} = \bar{z}\cdot\bar{w}$ $\qquad$ verifichiamo.

$z = x+iy, \quad w = a+ib$

$$z\cdot w = ax - yb + i(ya + xb)$$

$$\overline{z\cdot w} = ax - yb - i(ya + xb)$$

$$\bar{z}\cdot\bar{w} = (x-iy)\cdot(a-ib) = ax - by - i(ya + xb)$$

Modulo

$$|z| = \sqrt{x^2+y^2} \quad \text{se} \quad z = x+iy, \; x,y\in\mathbb{R}.$$

(*) $|z\cdot w| = |z|\cdot|w|$

Es $z = i + 2i\cdot i$ trovare $\operatorname{Re} z$, $\operatorname{Im} z$.

dimostriamolo:

$z = x+iy$, $w = a+ib$

$$z\cdot w = (ax-by) + i(ay+bx)$$

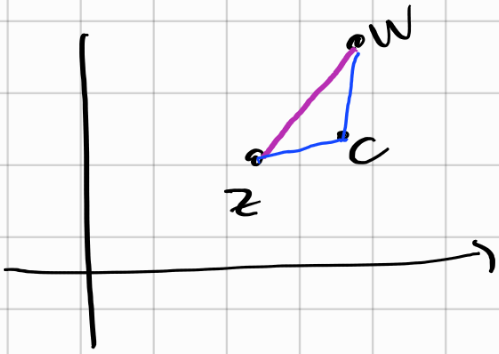
$$|z\cdot w| = \sqrt{(ax-by)^2 + (ay+bx)^2}$$

$$= \sqrt{a^2x^2 + b^2y^2 - \cancel{2axby} + a^2y^2 + b^2x^2 + \cancel{2aybx}}$$

$$= \sqrt{a^2(x^2+y^2) + b^2(y^2+x^2)} = \sqrt{(x^2+y^2)(a^2+b^2)}$$

$$= \sqrt{x^2+y^2}\cdot\sqrt{a^2+b^2} = |z|\cdot|w| \;\square$$

disug. triangolare



$$|z-w| \le |z-c| + |c-w| \iff |z+w| \le |z| + |w|$$

dimostrare!

• Dato z : $\exists\, w$: $w \cdot z = 1$.

$$z = x+iy$$

$$z \cdot \bar{z} = (x+iy)(x-iy) = x^2 - (iy)^2 = x^2+y^2$$

$$\rightarrow \quad |z|^2 = \underbrace{z \cdot \bar{z}}$$

$$z+\bar{z} = 2x$$

$$\begin{cases} \operatorname{Re} z = \dfrac{z+\bar{z}}{2} \\ \operatorname{Im} z = \dfrac{z-\bar{z}}{2i} \end{cases}$$

$$z-\bar{z} = 2iy$$

$$\sqrt{x^2+y^2} = 0 \iff \begin{cases} x=0 \\ y=0 \end{cases}$$

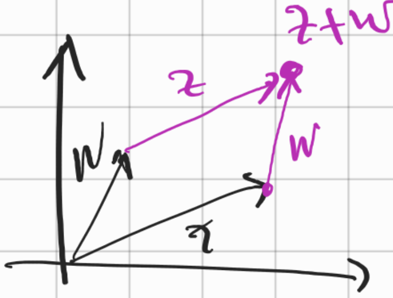
$\Updownarrow$

se $|z| \neq 0$ cioè $z \neq 0$

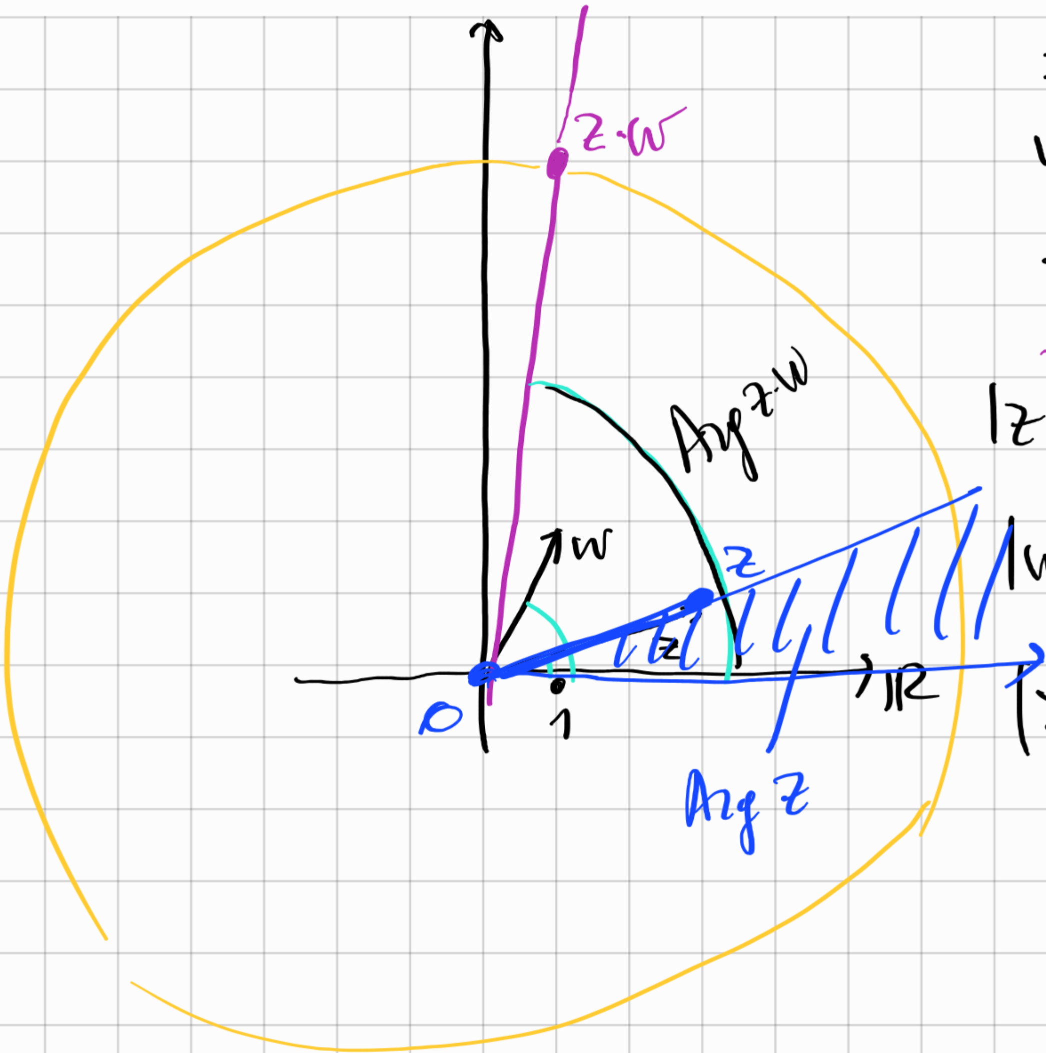
$$\frac{z \cdot \bar{z}}{|z|^2} = \frac{|z|^2}{|z|^2} = 1$$

$$w = \frac{1}{z} = z^{-1} = \frac{\bar{z}}{|z|^2}$$

$\mathbb{C}$ è un campo che estende $\mathbb{R}$



Come si interpreta geometricamente la moltiplicazione?



$z = 3 + i$

$w = 1 + 2i$

$z \cdot w = 1 + 7i$

$|z| = \sqrt{9+1} = \sqrt{10}$

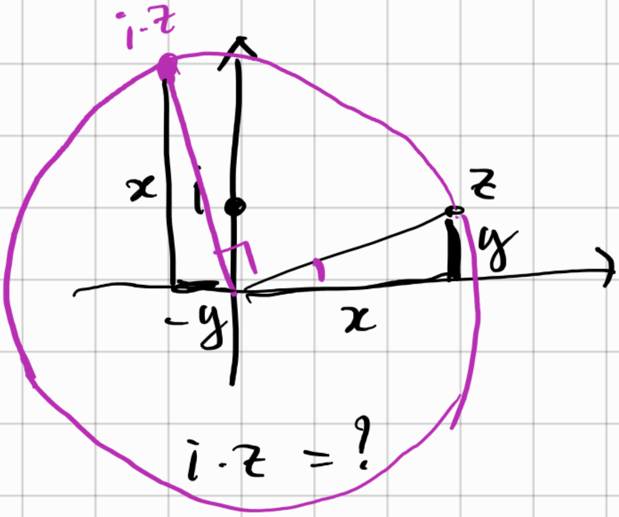
$|w| = \sqrt{1+4} = \sqrt{5}$

$|z \cdot w| = \sqrt{1+49}$
$= \sqrt{50}$

- $z \cdot w$ è quel numero complesso che ha modulo $|z \cdot w|$ il prodotto dei moduli $|z||w|$ e argomento la somma degli argomenti. *

Arg z = l'angolo individuato dalla semiretta uscente da O e passante per z con la semiretta dei reali positivi.

Es $\quad i$



$|i| = 1$

$\text{Arg } i =$ angolo retto

$z = x + iy \qquad i \cdot z = ?$

$|i \cdot z| = |i| \cdot |z| = |z|$

$iz = ix - y$

$= -y + ix$

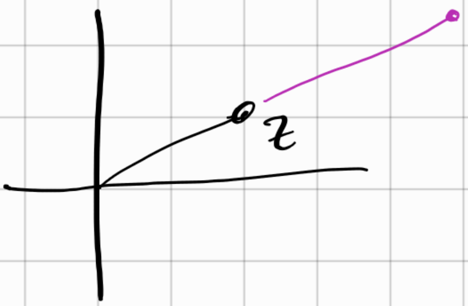
$\text{Arg}|iz| = \text{Arg } z + 90°$

In generale

$t \in \mathbb{R}, \quad z \in \mathbb{C} \qquad z = x + iy$

$t \cdot z = \begin{cases} tx + ity \\ \quad \parallel \\ (t + i \cdot 0) \cdot (x + iy) \end{cases}$

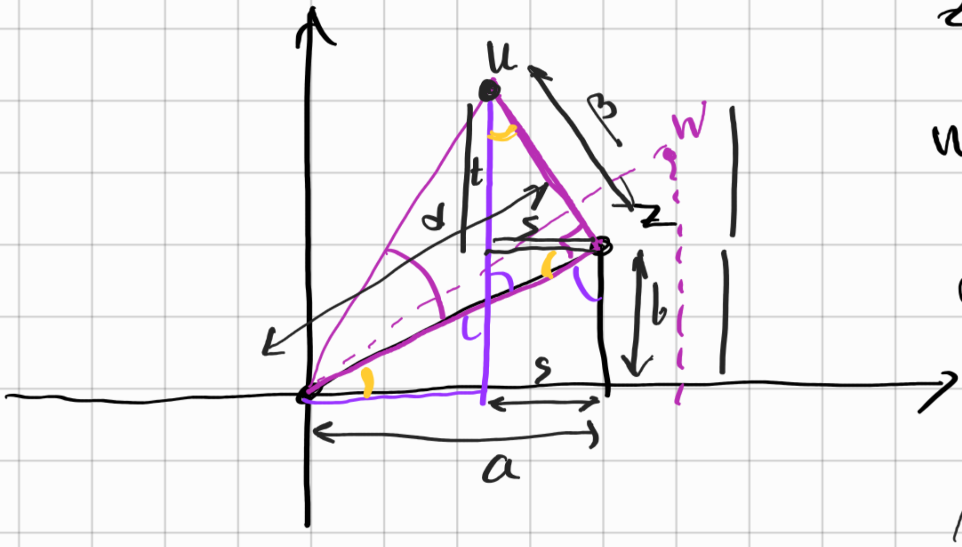
$\uparrow$



$\text{Arg}(tz) = \text{Arg } z \quad$ se $t \geq 0$

$(t \in \mathbb{R})$

Basta dimostrare (*) per numeri complessi con modulo che decido io



$z = a + ib$

$w = \alpha + i\beta$

$\alpha = |z|$

↑ ipotesi semplificativa.

Devo mostrare che u è multiplo di $z \cdot w$.

$$u = (a-s) + i(b+t)$$

$$\frac{s}{t} = \frac{b}{a} \qquad \frac{t}{\beta} = \frac{a}{\alpha} \qquad t = \frac{a\beta}{\alpha}$$

$$s = \frac{tb}{a} = \frac{a\beta b}{a\alpha}$$

$$u = \left(a - \frac{a\beta b}{a\alpha}\right) + i\left(b + \frac{a\beta}{\alpha}\right)$$

$$z \cdot w = (a+ib)\cdot(\alpha + i\beta)$$
$$= a\alpha - b\beta + i(b\alpha + a\beta)$$

$$= \frac{a^2\alpha - a\beta b}{a\alpha} + i\,\frac{a\alpha b + a^2\beta}{a\alpha}$$

$$= \frac{a\alpha - b\beta + i(b\alpha + a\beta)}{\alpha} = \frac{z \cdot w}{\alpha} \qquad \square$$

TOPOLOGIA di $\mathbb{C}$.

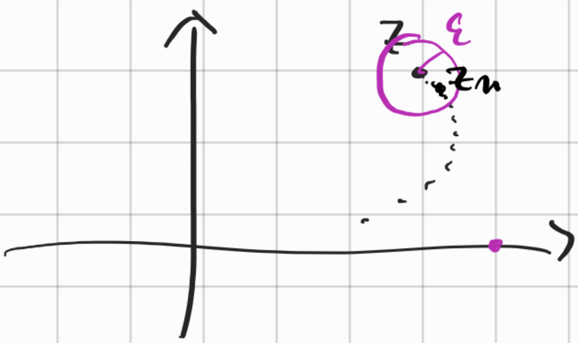
$z_n \in \mathbb{C}$, $z \in \mathbb{C}$ $\quad z_n \to z$?

definizioni equivalenti: $\quad z_n = x_n + i y_n$, $\quad z = x + iy$

① $|z_n - z| \to 0$

② $\begin{cases} x_n \to x \\ y_n \to y \end{cases}$ $\qquad \lim_{n \to +\infty} z_n = z$

③ $\forall \varepsilon > 0 \ \exists N : \forall n \in \mathbb{N} : n \geq N \Rightarrow |z_n - z| < \varepsilon$



$z_n \to \infty$ se $|z_n| \to +\infty$

$\bar{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$