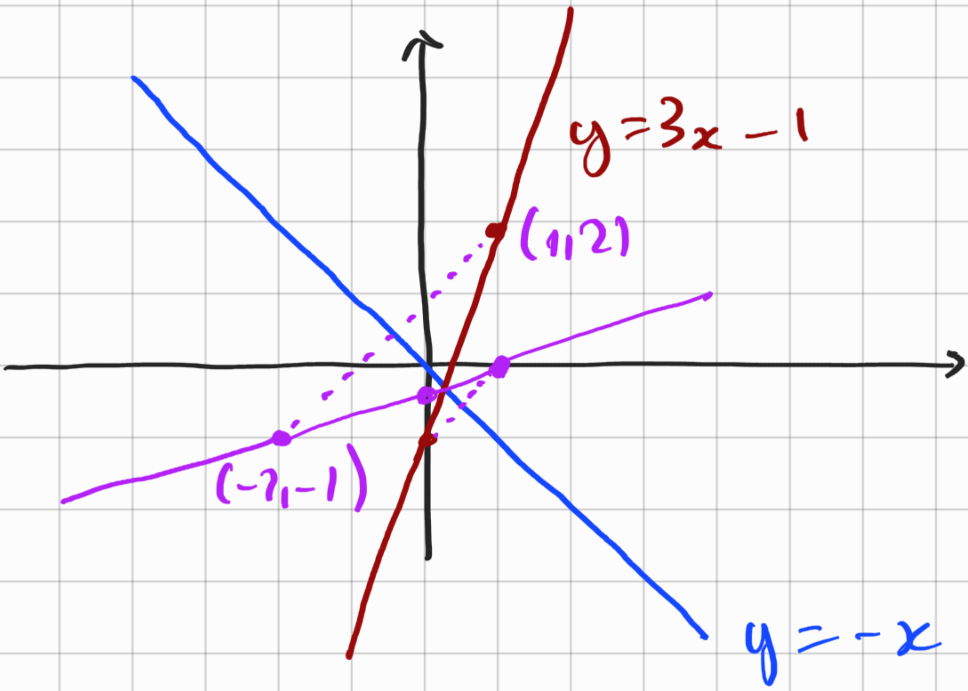
ANALISI MATEMATICA B
LEZIONE 16 - 25.10.2021

$$\min_{x\in\mathbb{R}} (x^2+x+1) = \min\{x^2+x+1 : x\in\mathbb{R}\}$$
$$= \min\{y\in\mathbb{R} : \exists x\in\mathbb{R} : x^2+x+1=y\}$$

$$g(x) = \max\{0, x^2-1\} = \begin{cases} 0 & \text{se } 0\geq x^2-1 \\ x^2-1 & \text{se } 0< x^2-1 \end{cases}$$



$$(x,y)\mapsto(-y,-x)$$

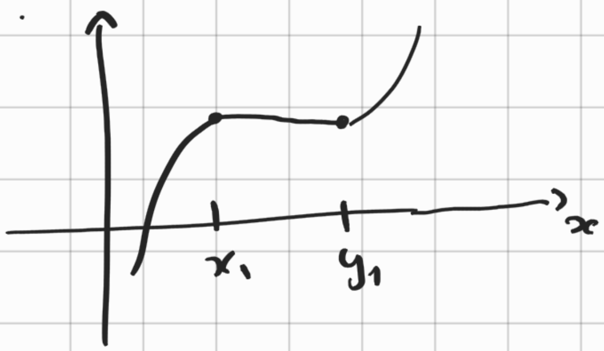
$$y=\frac{1}{3}x-\frac{1}{3}$$

Oss Sia f crescente ma non strett. crescente.

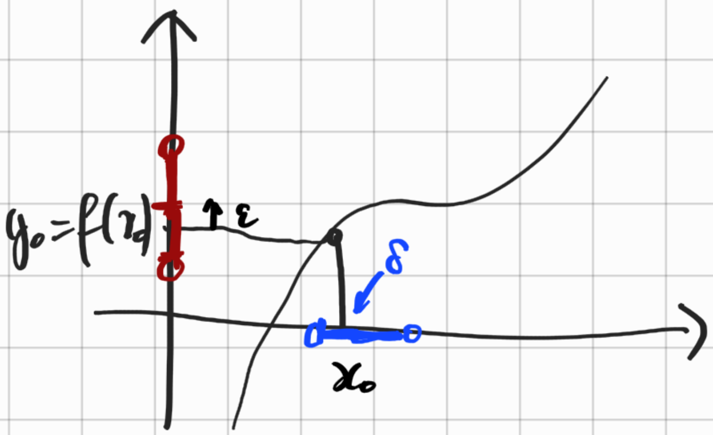
$\forall x<y \quad f(x)\leq f(y)$

$\exists x_1<y_1 \quad f(x_1)=f(y_1)$

$\forall x\in[x_1,y_1] \quad f(x)=f(x_1)$



FUNZIONE CONTINUA



$$y = f(x)$$

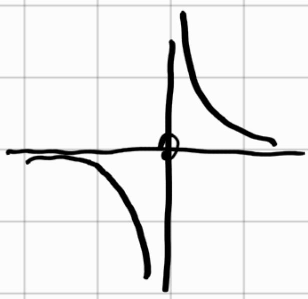
$$f : A \subseteq \mathbb{R} \to \mathbb{R}$$

Def Diremo che f è continua nel punto $x_0 \in A$

Se $\forall \varepsilon > 0 \ \exists \delta > 0 : |x - x_0| < \delta \implies |f(x) - f(x_0)| < \varepsilon$.

$$x_0 - \delta < x < x_0 + \delta$$

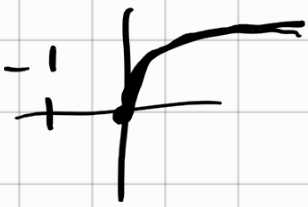
$$f(x) = \frac{1}{x}$$



$$f : \underbrace{\mathbb{R} \setminus \{0\}}_{} \to \mathbb{R} \qquad A = \mathbb{R} \setminus \{0\}$$

Def 2. $f : A \subseteq \mathbb{R} \to \mathbb{R}$ è continua se

f è continua in x_0 per ogni $x_0 \in A$.

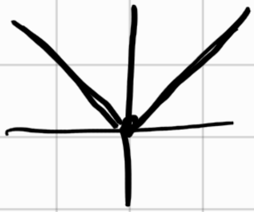


$$g(x) = \sqrt{x}$$

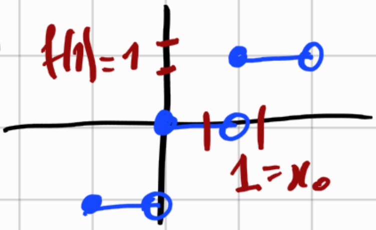
[**Esercizio** (*) Esiste $f : \mathbb{Q} \to \mathbb{R}$ continua
tale che se $g : \mathbb{R} \to \mathbb{R}$ "estende" f
($g(x) = f(x) \ \forall x \in \mathbb{Q}$) g non può essere continua]

Esempio $f(x) = |x|$

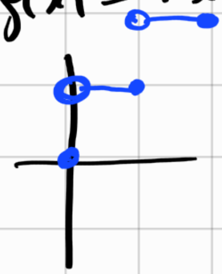
è continua.



Esempio $f(x) = \lfloor x \rfloor$



$g(x) = \lceil x \rceil$



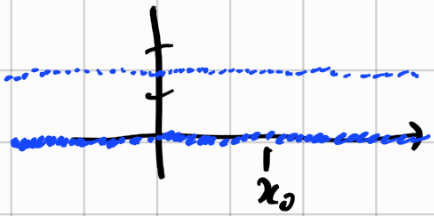
$f(x)$ non è continua in $x_0 = 1$.

$f : \mathbb{R} \to \mathbb{R}$

Esempio (fn. di Dirichlet) $f : \mathbb{R} \to \mathbb{R}$

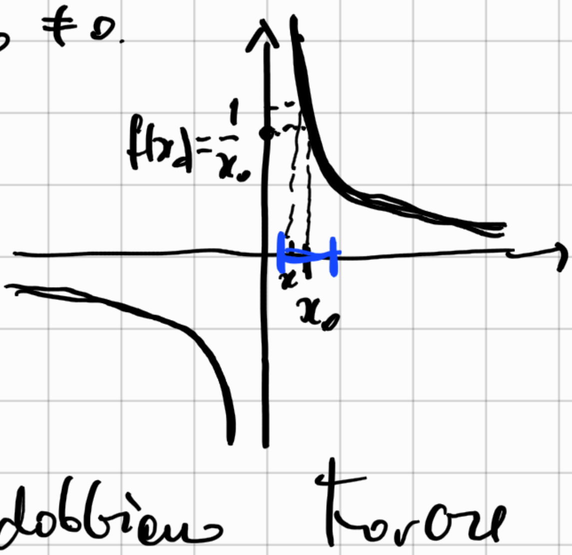
$$f(x) = \begin{cases} 1 & \text{se } x \in \mathbb{Q} \\ 0 & \text{se } x \notin \mathbb{Q} \end{cases}$$

f non è continua in nessun punto.



Esempio $f(x) = \frac{1}{x}$ è continua. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$

Dobbiamo mostrare che $f(x) = \frac{1}{x}$ è continua in x_0 per ogni $x_0 \neq 0$.



Dato $\varepsilon > 0$ dobbiamo trovare $\delta > 0$:

$$\left| \frac{1}{x} - \frac{1}{x_0} \right| \overset{?}{<} \varepsilon \qquad \left(x \ \underline{|x - x_0| < \delta} \right)$$

se $|x-x_0|<\delta \Rightarrow$

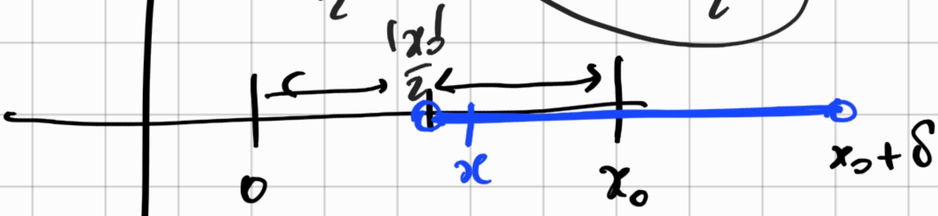
$$\left|\frac{x_0-x}{x\cdot x_0}\right| = \frac{|x-x_0|}{|x|\cdot|x_0|} < \frac{\delta}{|x|\cdot|x_0|} < \frac{\delta}{\frac{|x_0|}{2}\cdot|x_0|} = \frac{2\delta}{|x_0|^2} \le \varepsilon \quad ?*$$

$$\left[\ |a\cdot b| = |a|\cdot|b| \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|} \qquad |-x| = |x|\ \right]$$

$$|x_0| = |0-x_0| \le |0-x| + |x-x_0| < |x| + \frac{|x_0|}{2}$$

(1) Supponiamo $\delta < \frac{|x_0|}{2}$

se $|x-x_0| < \frac{|x_0|}{2} \Rightarrow |x| > \frac{|x_0|}{2}$



$$x_0 - \frac{x_0}{2} < x < x_0 + \frac{x_0}{2}, \qquad x_0 - \frac{x_0}{2} = \frac{x_0}{2}$$

(2) Scelgo $\delta \le \frac{\varepsilon}{2}|x_0|^2$

disuguaglianza triangolare: $|a-c| \le |a-b| + |b-c|$

conversità: $|a+b| \le |a| + |b|$

disug. triang. inversa: $|a-b| \ge \big|\,|a-c| - |b-c|\,\big|$.

$$\forall \varepsilon > 0 : \exists \delta > 0 : |x-x_0| < \delta \Rightarrow \left|\frac{1}{x} - \frac{1}{x_0}\right| < \varepsilon.$$

$$\delta = \min\left\{\frac{\varepsilon}{2}|x_0|^2,\ \frac{|x_0|}{4}\right\}$$

$\square$