

New Trends in Elliptic PDEs: remembering Sergio Campanato

Aula Magna at the Department of Mathematics, University of Pisa

Thursday, Mar 6, 2025 - Friday, Mar 7, 2025

THURSDAY 6 MARCH

14.30-14.40 Introduction

14.40-15.30 - **Cristiana De Filippis**: *μ -Ellipticity and nonautonomous integrals*

15.30- 16.20 - **Giuseppe Mingione**: TBA

16.20-17.00 - COFFEE BREAK

17.00-17.50 - **Alessandro Pigati**: *Diffuse improvement of flatness in codimension two*

17.50- 18.40 - **Riccardo Tione**: *On the Lawson-Osserman conjecture*

FRIDAY 7 MARCH

9.00-9.50 - **Vincenzo Vespri**: *Remarks on Sobolev-Morrey-Campanato Spaces defined on $C^{0,\gamma}$ domains*

9.50-10.40 - **Filomena Pacella**: *A shape optimization problem in cylinders and related questions*

10.40-11.10 - COFFEE BREAK

11.10-12.00 - **Alessandro Iacopetti**: *The Prescribed Mean Curvature Equation in the Lorentz-Minkowski Space: New Regularity Results and Open Problems*

12.00-12.50 - **Piermarco Cannarsa**: *Domain invariance for nonlinear diffusion models*

ABSTRACTS

Cristiana De Filippis, Università di Parma

Title: *μ -ellipticity and nonautonomous integrals*

Abstract: μ -ellipticity is a degenerate form of ellipticity, typical of fundamental integrals in geometric analysis or nonlinear elasticity. The regularity theory for autonomous models, e. g., the area functional, is classical after Bombieri and De Giorgi and Miranda (ARMA '69), while the presence of external ingredients may lead to disturbing anomalies, cf. Ladyzhenskaya and Ural'tseva (CPAM '70) and Giaquinta and Modica and Souček (Comm. Univ. Carolinae '79). I will discuss recent advances on nonautonomous, μ -elliptic problems focusing on sharp results and borderline configurations for the validity of Schauder theory. From recent, joint work with Filomena De Filippis (Parma), Giuseppe Mingione (Parma) and Mirco Piccinini (Pisa).

Giuseppe Mingione, Università di Parma

TBA

Alessandro Pigati, Università Bocconi

Title: *Diffuse improvement of flatness in codimension two*

Abstract: The Allen-Cahn energy is by now a well-understood way to approximate the area functional for hypersurfaces. Critical points of it converge to minimal hypersurfaces as we send the scaling parameter to zero, and the same holds for the gradient flow. Inspired by this parallel, De Giorgi proposed a conjecture which is analogous to the Bernstein problem for minimal graphs: given an entire critical point in dimension $n < 9$, monotone in one direction, is it necessarily a function of just one coordinate?

Savin solved this conjecture assuming local minimality, which can be seen to be implied by a mild additional assumption. We present an analogue in codimension two, for the abelian Yang-Mills-Higgs energy, which is known to approximate area in codimension two. The result is based on an improvement of flatness in the style of Allard and is partly inspired by an alternative proof of Savin's theorem by Wang. It also uses recent stability results in dimension two by Halavati. We also discuss some open questions. This is joint work with Guido De Philippis (NYU Courant) and Aria Halavati (NYU Courant).

Riccardo Tione, Università di Torino

Title: *On the Lawson-Osserman conjecture*

Abstract: In 1977, H.B. Lawson and R. Osserman conjectured that Lipschitz maps which are critical with respect to outer variations of the area functional are also critical with respect to domain variations. In this talk I will present a solution to this conjecture in the planar case. This result was obtained in collaboration with J. Hirsch and C. Mooney.

Vincenzo Vespri, Università di Firenze

Title: *Remarks on Sobolev-Morrey-Campanato Spaces defined on $C^{0,\gamma}$ domains*

Abstract: We discuss a few old results concerning embedding theorems for Campanato and Sobolev-Morrey spaces adapting the formulations to the case of domains of class $C^{0,\gamma}$, and we present more recent results concerning the extension of functions from Sobolev-Morrey spaces defined on those domains. As a corollary of the extension theorem we obtain an embedding theorem for Sobolev-Morrey spaces on arbitrary $C^{0,\gamma}$ domains.

Filomena Pacella, Università di Roma La Sapienza

Title: *A shape optimization problem in cylinders and related questions*

Abstract: We present some results about a shape optimization problem for the torsional energy associated to domains contained in an infinite cylinder, under a volume constraint. We prove that a minimizer exists for all fixed volumes, and we show some of its geometric properties. Then the question is to understand whether it is the "trivial" domain given by a bounded cylinder, whose corresponding torsion function depends only on one variable. By studying the second variation of the energy functional we are able to prove that this is not always the case. Indeed, the bounded cylinder can become unstable, which means that functions with flat level sets are not always the best candidates for optimizing the energy. As the shape optimization problem is related to the variational formulation of a corresponding overdetermined problem we also deduce results for this problem. On the contrary differences with the question of characterizing constant mean curvature surfaces intersecting orthogonally a cylinder will appear. The results presented are contained in papers in collaboration with D.Afonso, P.Caldirola, A.Iacopetti, D.Ruiz and P. Sicbaldi.

Alessandro Iacopetti, Università di Torino

Title: *The Prescribed Mean Curvature Equation in the Lorentz-Minkowski Space: New Regularity Results and Open Problems*

Abstract: In this talk, we present recent results on the regularity of the minimizer of the Born-Infeld energy, in relation to the existence of weak solutions for the prescribed mean curvature equation in the Lorentz-Minkowski space for spacelike hypersurfaces, when the mean curvature belongs to L^p .

In the first part of the talk, we will show a new gradient estimate for entire smooth solutions of the prescribed mean curvature equation. We will then prove that if p is strictly greater than the dimension N , the unique minimizer of the Born-Infeld energy, which is a priori only Lipschitz continuous, is in fact a strictly spacelike weak solution of the equation and belongs to $W_{loc}^{2,p}$. Finally, we will show some new results and discuss open problems related to the case of bounded domains.

Piermarco Cannarsa, Università di Roma Tor Vergata

Title: *Domain invariance for nonlinear diffusion models*

Abstract: [An optimal partition problem for eigenvalues, Journal of scientific Computing 31 (2007), 1-2, pp. 5–18], Caffarelli and Lin constructed a bilinear control that keeps the norm of the linear heat flow on a bounded domain constant in time. The construction consists of taking a suitable nonlocal feedback: this problem has therefore clear connections with domain invariance for nonlinear diffusion equations with nonlocal terms.

In this talk, we will discuss abstract evolution equations in Hilbert and Banach spaces, for which we will give necessary and sufficient conditions for the invariance of a space domain. Then, we will apply these results to concrete PDE models that generalize the one studied by Caffarelli and Lin. In particular, we will show how the addition of nonlocal terms, allows to force the flow of nonlinear heat equations and the Navier-Stokes system to remain in specific space domains such as a sphere or a hyperplane in the configuration space.