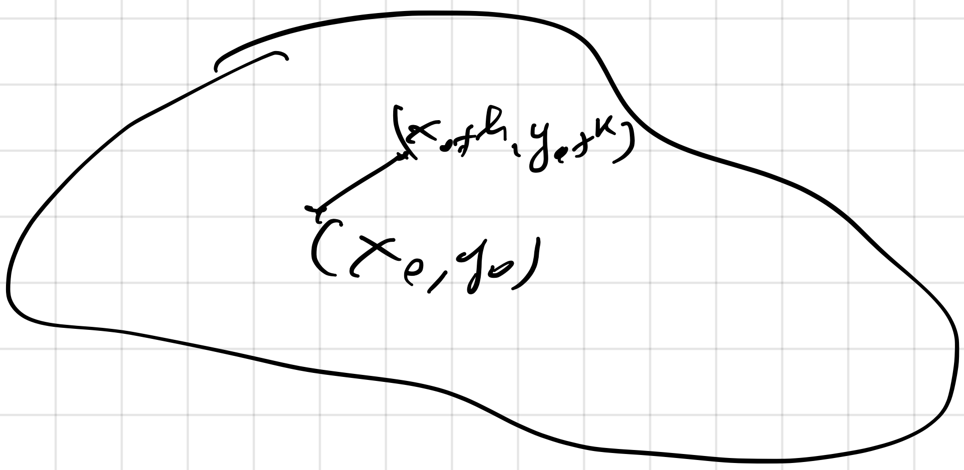
DIFFERENZIABILITÀ

f diff. in (x_0, y_0) se e solo se

$\exists\, \alpha, \beta \in \mathbb{R}$ t.c.

$$\lim_{(h,k)\to(0,0)} \frac{f(x_0+h, y_0+k) - f(x_0, y_0) - \alpha h - \beta k}{\sqrt{h^2+k^2}} = 0$$



Teo. f diff in (x_0, y_0) implica:

i) $\exists\ \nabla f(x_0, y_0)$, $\frac{\partial f}{\partial x}(x_0,y_0)$, $\frac{\partial f}{\partial y}(x_0,y_0)$

ii) $\alpha = \frac{\partial f}{\partial x}(x_0,y_0)$ e $\beta = \frac{\partial f}{\partial y}(x_0,y_0)$

Dim.

$$\boxed{f(x_0+h, y_0+k) - f(x_0,y_0) - \alpha h - \beta k = o(\sqrt{h^2+k^2})}\ Hp$$

Come provare che $\exists\ \frac{\partial f}{\partial x}(x_0,y_0)$ e vale α!

$$\boxed{\frac{\partial f}{\partial x}(x_0,y_0)} = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h} =$$

$$\overset{(Hp)\ \boxed{k=0}}{=} \lim_{h\to 0} \frac{\alpha h + o(\sqrt{h^2})}{h} = \lim_{h\to 0}\left(\alpha + \underbrace{\frac{o(|h|)}{h}}_{0}\right) = \boxed{\alpha}$$

Stesso discorso per $\boxed{\frac{\partial f}{\partial y}(x_0,y_0) = \beta}$

$\blacksquare$

DOMANDA $f(x,y) = x^2 y^3$

In quali punti $(x_0, y_0) \in \mathbb{R}^2$ è diff. f?

1) Continua ovunque

2) $\nabla f(x_0, y_0)$ $\frac{\partial f}{\partial x}(x,y) = 2xy^3$, $\frac{\partial f}{\partial y}(x,y) = 3x^2y^2$

$\frac{\partial f}{\partial x}(x_0, y_0) = 2x_0 y_0^3$, $\frac{\partial f}{\partial y}(x_0, y_0) = 3x_0^2 y_0^2$

3) Per quali (x_0, y_0) si ha la seguente proprietà:

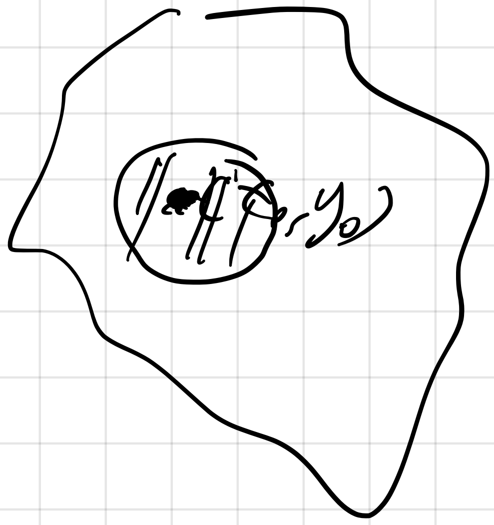
$$\lim_{(h,k) \to (0,0)} \frac{(x_0+h)^2 (y_0+k)^3 - x_0^2 y_0^3 - 2x_0 y_0^3 h - 3x_0^2 y_0^2 k}{\sqrt{h^2+k^2}}$$

$$= 0$$

Teo (del diff. totale)

$f: \Omega \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$, $(x_0, y_0) \in \Omega$

Supponiamo:



i) $\exists \, \nabla f(x,y)$ per $(x,y) \in B_\delta(x_0,y_0)$ con $\delta > 0$.

ii) $\nabla f(x,y)$ è continuo in (x_0,y_0)

$$\lim_{(x,y)\to(x_0,y_0)} \frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}(x_0,y_0)$$

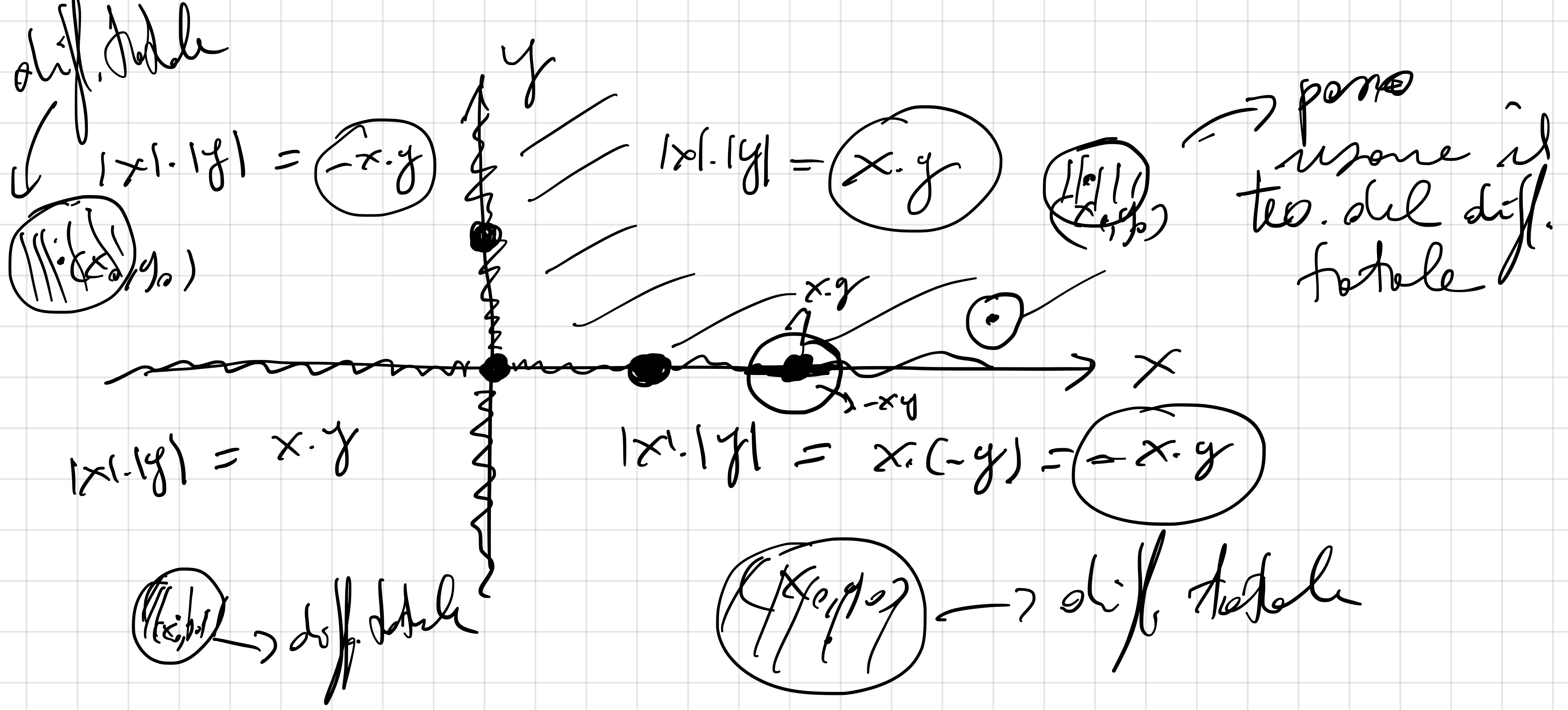
$$\lim_{(x,y)\to(x_0,y_0)} \frac{\partial f}{\partial y} = \frac{\partial f}{\partial y}(x_0,y_0)$$

Tesi La funzione f è diff. in (x_0,y_0)

∎

Esercizio Dire in quali punti di $\mathbb{R}^2$ è differenziabile la funzione

$$f(x,y) = |x| \cdot |y|$$



Per il teo del diff. totale $f(x,y)$ è diff. su $\mathbb{R}^2 \setminus$ gli assi.

Cosa succede negli assi?
Per esempio in $(0,0)$?

$\exists \nabla f(0,0)$

$$\frac{\partial f}{\partial x}(0,0) \overset{?}{=} \lim_{h\to 0} \frac{\overset{0}{\overset{\shortparallel}{f(h,0)}} - \overset{0}{\overset{\shortparallel}{f(0,0)}}}{h} = \boxed{0}$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{k\to 0} \frac{\overset{0}{\overset{\shortparallel}{f(0,k)}} - \overset{0}{\overset{\shortparallel}{f(0,0)}}}{k} = \boxed{0}$$

$$\lim_{(h,k)\to(0,0)} \frac{|h|\cdot|k|}{\sqrt{h^2+k^2}} \overset{?}{=} \boxed{0}$$

se sì $\Rightarrow$ diff.-le $(0,0)$

se no $\Rightarrow$ non diff.-le $(0,0)$

$\downarrow$ le coordinate polari

0

$$\frac{\rho^2 |\cos\theta|\,|\sin\theta|}{\rho} = \rho\, |\cos\theta|\,|\sin\theta| \leq \rho \xrightarrow{\rho\to 0} 0$$

Cosa succede sugli assi escluso (0,0)?

$(x_0, 0)$ e $(0, y_0)$
$x_0 \neq 0$, $y_0 \neq 0$

Studiamo se è lì diff. in $(x_0, 0)$ con $x_0 \neq 0$.

1) Studiamo $\nabla f(x_0, 0)$.

$$\partial_x f(x_0,0) = \lim_{h \to 0} \frac{\overset{0}{f(x_0+h,0)} - \overset{0}{f(x_0,0)}}{h} = \boxed{0}$$

$$\partial_y f(x_0,0) = \lim_{k \to 0} \frac{f(x_0,k) - \overset{0}{f(x_0,0)}}{k} = \nexists$$

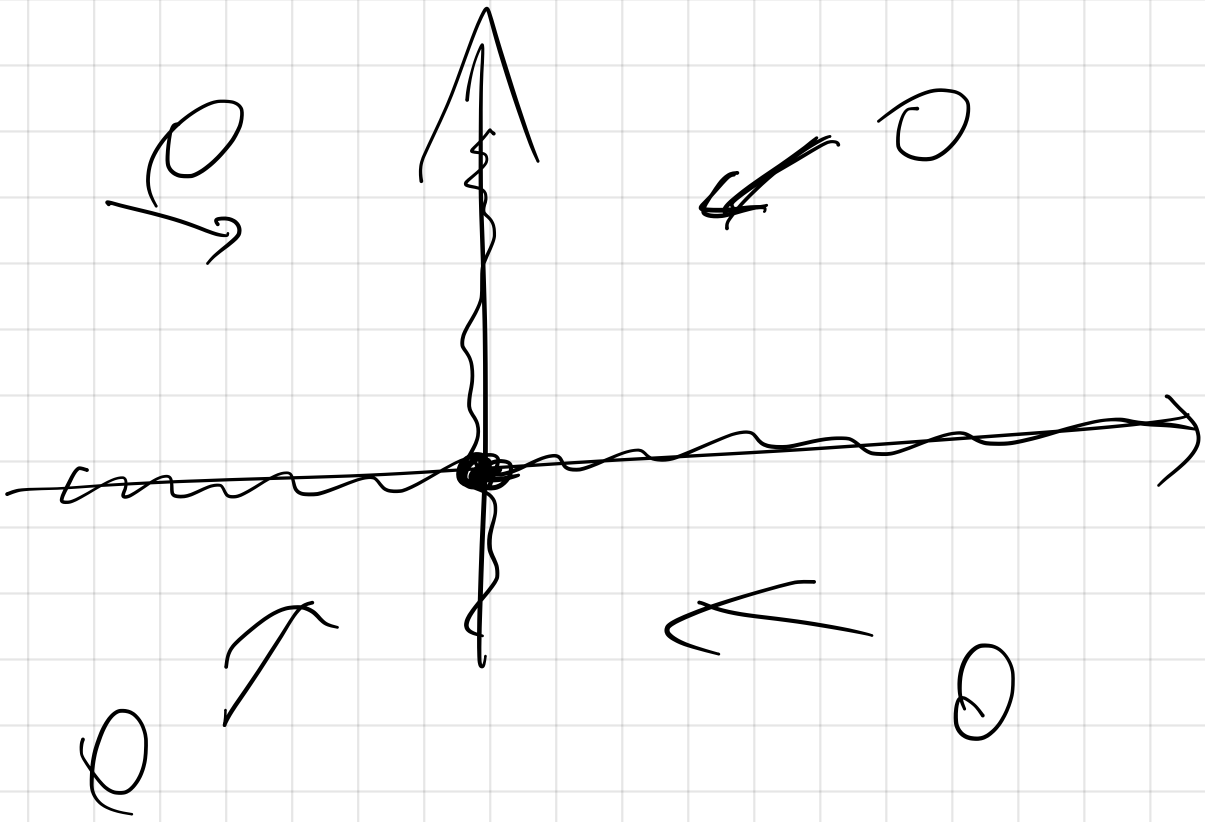
$$\lim_{k \to 0} \left(\frac{|x_0| \cdot |k|}{k} \right) = |x_0| \lim_{k \to 0} \frac{|k|}{k} \nexists$$

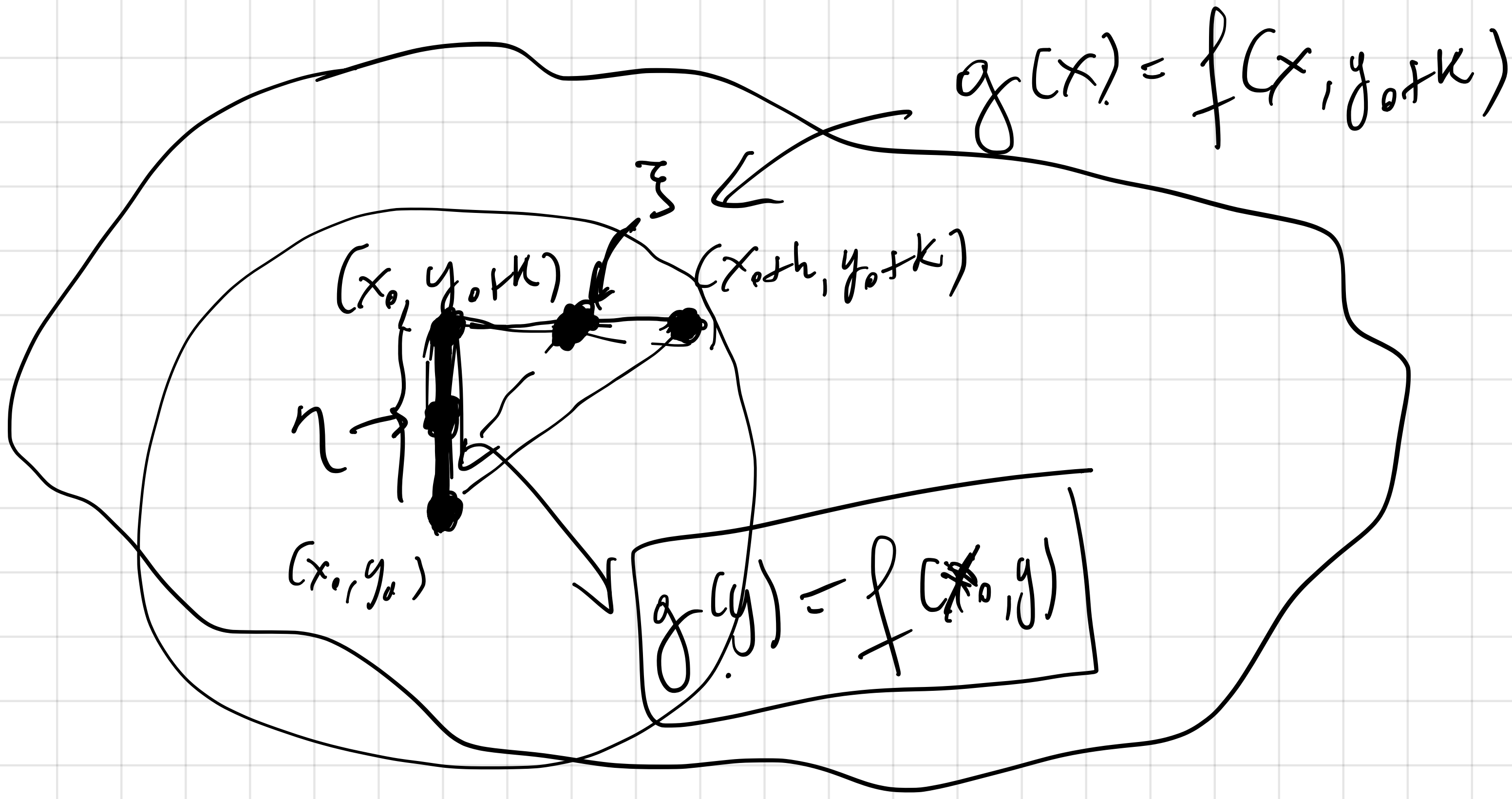
perché

$$\lim_{k \to 0^+} \frac{|k|}{k} = \boxed{1} \text{ e } \lim_{k \to 0^-} \frac{|k|}{k} = \boxed{-1}$$
$\neq$

Unendo tutte le informazioni abbiamo che $f(x,y) = |x| \cdot |y|$ è diff. in

$$\boxed{\{(x,y) \in \mathbb{R}^2 \mid x \cdot y \neq 0\} \cup \{(0,0)\}}$$





Tesi:

$$\boxed{\lim_{(h,k)\to(0,0)} \frac{f(x_0+h, y_0+k) - f(x_0, y_0) - \frac{\partial f}{\partial x}(x_0,y_0)h - \frac{\partial f}{\partial y}(x_0,y_0)k}{\sqrt{h^2+k^2}} = 0}$$

$$f(x_0+h, y_0+k) - f(x_0, y_0) =$$

$$f(x_0+h, y_0+k) - f(x_0, y_0+k)]$$

$$+ f(x_0, y_0+k) - f(x_0, y_0)$$

Teo. del valor medio

$g : (a,b) \longrightarrow \mathbb{R}$

$$\boxed{\frac{g(b) - g(a)}{b-a} = g'(\xi)}, \quad \xi \in (a,b)$$

Per questo teo. abbiamo

$$\boxed{\begin{aligned} &\frac{f(x_0+h, y_0+k) - f(x_0, y_0+k)}{h} = \\ &= \partial_x f(\xi, y_0+k) \qquad \xi \in [x_0, x_0+h] \end{aligned}}$$

$$\boxed{\frac{f\overbrace{(x_0, y_0+k)}^{b} - f\overbrace{(x_0, y_0)}^{a}}{k = \boxed{b-a}} = \partial_y f(x_0, \eta) \qquad \eta \in [\underbrace{y_0}_{a}, \underbrace{y_0+k}_{b}]}$$

$$f(x_0+h, y_0+k) - f(x_0, y_0+k) = h\, \partial_x f(\xi, y_0+k)$$

$$f(x_0, y_0+k) - f(x_0, y_0) = k\, \partial_y f(x_0, \eta)$$

Sommando

$$\boxed{\begin{aligned} f(x_0+h, y_0+k) - f(x_0, y_0) = {} & h\, \partial_x f(\xi, y_0+k) \\ & + k\, \partial_y f(x_0, \eta) \end{aligned}}$$

Quindi:

$$f(x_0+h, y_0+k) - f(x_0, y_0) - \partial_x f(x_0, y_0)\, h - \partial_y f(x_0, y_0)\, k$$

$$= \boxed{\begin{aligned} & h\, \partial_x f(\xi, y_0+k) + k\, \partial_y f(x_0, \eta) \\ & - \partial_x f(x_0, y_0)\, h - \partial_y f(x_0, y_0)\, k \end{aligned}}$$

Da provare che:

$$\lim_{(h,k)\to(0,0)} \frac{h\left[\partial_x f(\xi, y_0+k) - \partial_x f(x_0,y_0)\right] + k\left[\partial_y f(x_0,\eta) - \partial_y f(x_0,y_0)\right]}{\sqrt{h^2+k^2}}$$

Dico che

$$\lim_{(h,k)\to(0,0)} \underbrace{\frac{|h|}{\sqrt{h^2+k^2}}}_{\leq 1} \underbrace{\left[\partial_x f(\xi, y_0+k) - \partial_x f(x_0,y_0)\right]}_{\to 0 \text{ per (Hp)}}$$

Se $(h,k)\to(0,0)$
$\Downarrow$
$(\xi, y_0+k)\to(x_0,y_0)$

$$= \boxed{0}$$

Idem per

$$\lim_{(h,k)\to(0,0)} \underbrace{\frac{|k|}{\sqrt{h^2+k^2}}}_{\leq 1} \underbrace{\left[\partial_y f(x_0,\eta) - \partial_y f(x_0,y_0)\right]}_{\to 0 \text{ per (Hp)}} = \boxed{0}$$

$\boxtimes$

CALCOLO PIANO TANGENTE

Se $f : \Omega \xrightarrow{\subseteq \mathbb{R}^2} \mathbb{R}$ è diff. in (x_0, y_0) come si scrive l'eq. del piano tangente?

Il piano tangente in $\mathbb{R}^3$

$$\boxed{\alpha x + \beta y + \gamma z = \delta}$$

$$\boxed{z = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)}$$

$\alpha = \frac{\partial f}{\partial x}(x_0,y_0)$, $\beta = \frac{\partial f}{\partial y}(x_0,y_0)$, $\gamma = -1$

$\delta = -f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\,x_0 + \frac{\partial f}{\partial y}(x_0,y_0)\,y_0$

$$\lim_{(x,y)\to(0,0)} \frac{x^5 - y^4}{|x|^2 + |y|} \cdot \cancel{\lg\left(\frac{1}{x^2+y^2}\right)}$$

NON C'E'

$$\left|\frac{\rho^5\cos^5\theta - \rho^4\sin^4\theta}{\rho^2\cos^2\theta + \rho|\sin\theta|}\right| =$$

$$= \left|\frac{\rho^{\cancel{4}\,3}(\rho\cos^5\theta - \sin^4\theta)}{\cancel{\rho}\,(\rho\cos^2\theta + |\sin\theta|)}\right| \le g(\rho) \xrightarrow{\rho\to 0} 0$$

$$=$$

$$\left|\rho^3\,\frac{(\rho\cos^5\theta - \sin^4\theta)}{\rho\cos^2\theta + |\sin\theta|}\right|$$

$$\left|\frac{\rho^3\cdot\left(\left(\rho\cos^5\alpha-\operatorname{sen}^4\alpha\right)\right)}{\rho\cos^2\alpha+|\operatorname{sen}\alpha|}\right|$$

$$\leq \qquad \leq$$

$$\frac{|\rho|^{\cancel{3}\,2}\,|\rho+1|}{\rho\cos^2\alpha}=\frac{\rho^2\,|1+\rho|}{\cancel{\cos^2\alpha}}\ \overset{?}{\leq} g(\rho)$$

$\to 0$ $\to 0$

$$=\frac{\rho^3}{\rho}\cdot\frac{|\rho\cos^5\alpha-\operatorname{sen}^4\alpha|}{|\cos^2\alpha|+|\operatorname{sen}\alpha|}$$

$$\leq\frac{\rho^3\,\big|\,|\rho|\,|\cos\alpha|^5+|\operatorname{sen}\alpha|^4\big|}{\rho\cos^2\alpha}\leq\frac{\rho^3|\rho+1|}{\rho\cos^2\alpha}$$

$$\frac{x^5 - y^4}{x^2+|y|} = \frac{x^5}{x^2+|y|} - \frac{y^4}{x^2+|y|}$$

$$\frac{x^5}{x^2+|y|} \longrightarrow l_1 = 0 \qquad \frac{y^4}{x^2+|y|} \longrightarrow l_2 = 0$$

$$0 \leq \left| \frac{x^5}{x^2+|y|} \right| \leq \frac{|x|^5}{x^2+|y|} \leq \frac{|x|^5}{x^2} = |x|^3 \longrightarrow 0$$

$$0 \leq \left| \frac{y^4}{x^2+|y|} \right| \leq \frac{y^4}{x^2+|y|} \leq$$

$$\leq \frac{y^4}{|y|} = |y|^3 \longrightarrow 0$$

Cosa fare se ho lg?

$$\left(\frac{x^5-y^4}{x^2+|y|}\right)\lg\left(\frac{1}{x^2+y^2}\right)$$

$$\begin{cases} \dfrac{x^5}{x^2+|y|}\cdot\lg\left(\dfrac{1}{x^2+y^2}\right) \longrightarrow l_1=0 \\ \dfrac{y^4}{x^2+|y|}\lg\left(\dfrac{1}{x^2+y^2}\right) \longrightarrow l_2=0 \end{cases}$$

$\to |x|^3$

$0 \cdot +\infty$

$\to 0$?

$|y|^3 \to 0 \cdot \infty$

$$\left|\frac{x^5}{x^2+|y|}\lg\left(\frac{1}{x^2+y^2}\right)\right| \longrightarrow 0$$

$$0 \le \left|\frac{x^5}{x^2+|y|}\lg\left(\frac{1}{x^2+y^2}\right)\right| \le$$

$$\le \frac{|x|^5}{x^2+|y|}\left|\lg\left(\frac{1}{x^2+y^2}\right)\right|$$

$$\le \frac{|x|^5}{x^2}\left|\lg\left(\frac{1}{x^2+y^2}\right)\right|$$

$$= |x|^3\left|\lg\left(\frac{1}{x^2+y^2}\right)\right| \xrightarrow[(x,y)\to(0,0)]{?} 0$$

$$|x|^3 \left| \lg \left(\frac{1}{x^2+y^2} \right) \right| \xrightarrow{(x,y)\to(0,0)} 0 \;?$$

POLARI

$$|\rho|^3 |\cos\theta|^3 \left| \lg \left(\frac{1}{\rho^2} \right) \right| \leq$$

$$\leq 1 \cdot |\rho|^3 \, |\lg \rho^2| = \rho^3 \cdot 2 |\lg \rho| \xrightarrow{\rho \to 0} 0$$

ℓ

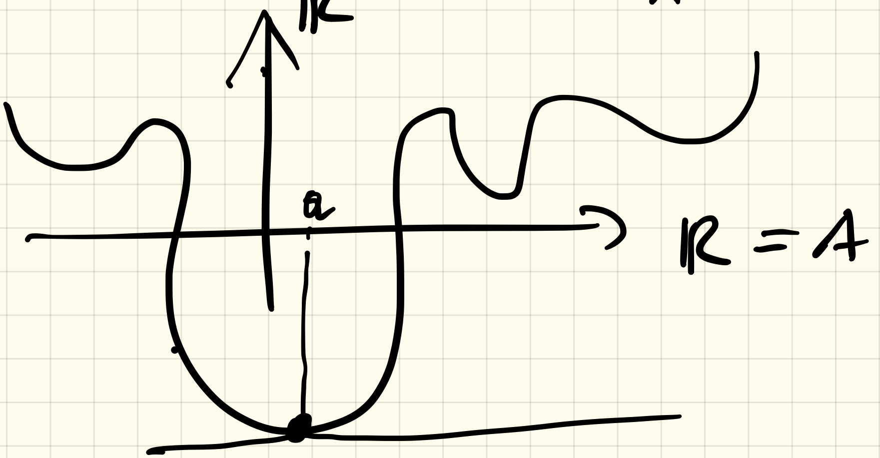
$\alpha > 0$ a base qualsiasi.

$$\lim_{x \to 0} x^{\alpha} \left(\log_a x\right) \longrightarrow 0$$

MAX E MIN ASSOLUTI

$f : A \longmapsto \mathbb{R}$

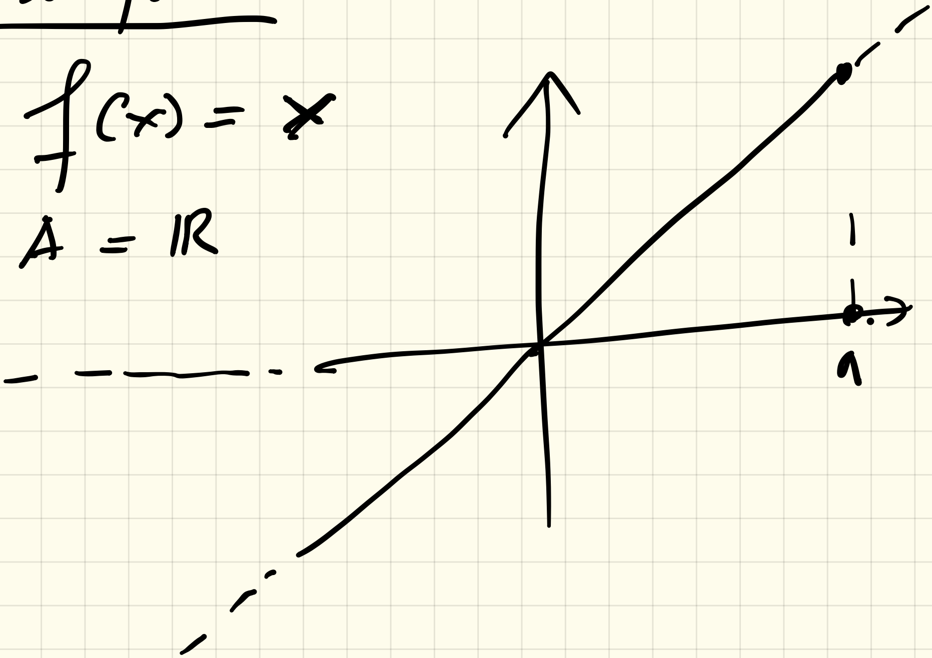
Def. Diciamo che f ammette <u>minimo</u> (assoluto) su A se $\exists\, a \in A$ t.c. $f(b) \geq f(a)$ $\forall b \in A$.



Esempio

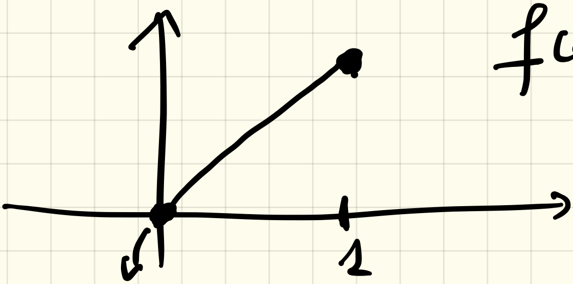
$f(x) = x$

$A = \mathbb{R}$



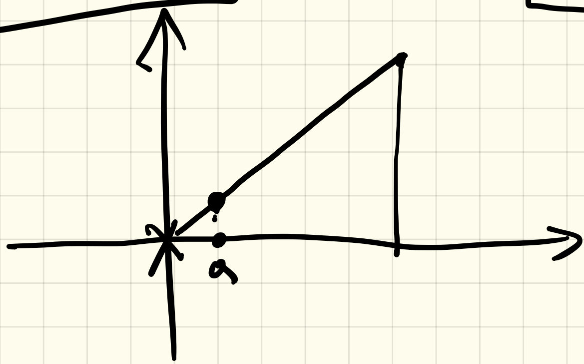
NON AMMETTE MAX E MIN

$f(x) = x \quad , \quad A = [0,1]$



$f(0) \leq f(x)$
$\forall x \in [0,1]$

$\underbrace{f(x) = x}$ $\quad A = \underbrace{(0,1]}$



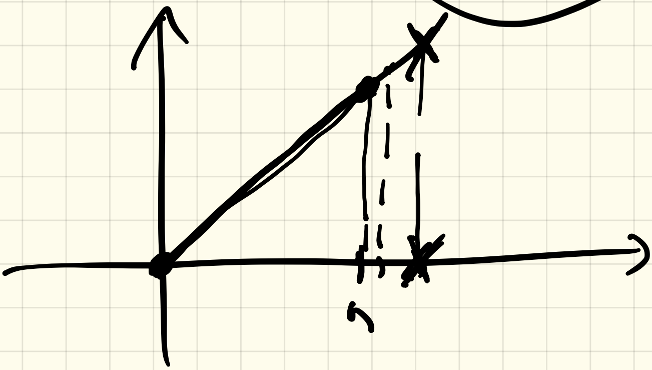
0 non può essere di minimo poiché $0 \notin A$.

Def. $f: A \longrightarrow \mathbb{R}$ diciamo che $a \in A$ è di max (assoluto) se

$$f(a) \geq f(b) \quad \forall b \in A$$

$f: \underset{\smile}{A} \longrightarrow \mathbb{R}$

$f(x) = x \qquad A = \underbrace{[0, 1)}$



<u>non</u> ammette <u>massimo assoluto</u>.

TEO WEIERSTRASS ANALISI 1

$f : [a, b] \longrightarrow \mathbb{R}$

$-\infty < a < b < +\infty$

$[a, b]$ limitato e chiuso
↓
include gli estremi

Se inoltre $\boxed{f \text{ è continua}}$

$\boxed{\Longrightarrow}$ $\exists$ Max e Min assoluti

ossia $\exists\ c, d \in [a, b]$

t.c.

$$f(c) \geq f(x) \quad \forall x \in [a, b]$$

$$f(d) \leq f(x) \quad \forall x \in [a, b]$$

Se esistono max e min <u>assoluti</u> come li troviamo concretamente?

$\boxed{\text{ALGORITMO}}$

- $f'(x) = 0$

Supponiamo di saper trovare gli zeri di $f'(x)$.

$\{\underbrace{x_0, x_1, \dots, x_k}_{\text{e sol.}}\}$ siano tutti e sol. i punti in cui si annulla f'.

- Considero i valori di f in $\{x_0, x_1, \dots, x_k\}$

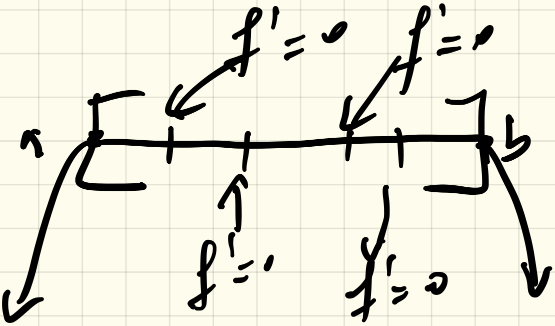
$\{\underline{f(x_0), f(x_1), \dots, f(x_k)}\}$

$$f(x) = x \quad , \quad [0,1]$$

$$f'(x) = 1 \rightsquigarrow \text{non si annulla mai!}$$

Quindi non basta solo considerare i punti dove si annulla f' per trovare max e min, ma bisogna anche prendere in considerazione gli estremi 0, 1.

- Considero i valori negli estremi $\{f(a), f(b)\}$
- Max f sarà il più grande tra i valori $\{f(x_0), \dots, f(x_k), \boxed{f(a), f(b)}\}$ e Min f sarà il più piccolo tra i valori $\{f(x_0), \dots, f(x_k), f(a), f(b)\}$



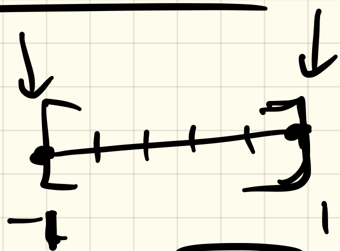
vanno considerati separatamente

$f(x) = x^2$, $[-1,1]$

$\underset{[-1,1]}{\mathrm{Max}} f$ e $\underset{[-1,1]}{\mathrm{Min}} f$?

Per Weierstrass Max e Min esistono.

$\underset{1}{\overset{\|}{f(-1)}}, \underset{1}{\overset{\|}{f(1)}}$



$f'(x) = 0 \iff 2x = 0 \implies \boxed{x=0}$

$\{f(-1), f(1), f(0)\} = \underline{\quad\quad}$

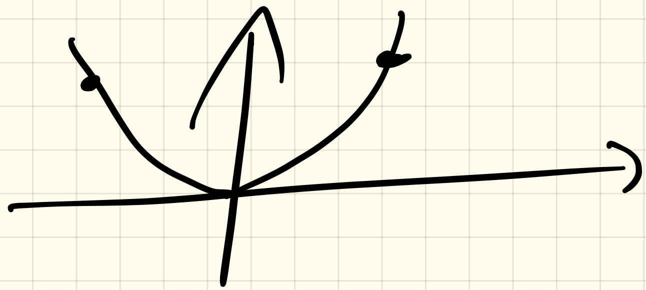
$= \{1, 1, 0\}$

Valore di max 1 e

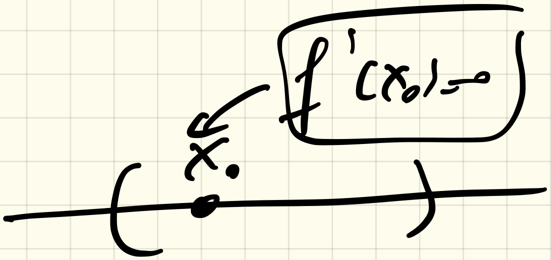
i punti di max sono $-1, 1$

Valore di min 0 e il

punto di min è $x = 0$



- Imporre $f'(x)=0$
- Tenere conto dei valori di f su a e b (gli estremi)



TEO. WEIERSTRASS IN PIÙ VARIABILI

$f : \Omega \longrightarrow \mathbb{R}$

$\Omega \subseteq \mathbb{R}^n$, $(\Omega \subseteq \mathbb{R}^2)$

sotto quali ipotesi

$\exists$ max e min ?

Def. Se $f : \underset{\smile}{A} \longrightarrow \mathbb{R}$ è tale

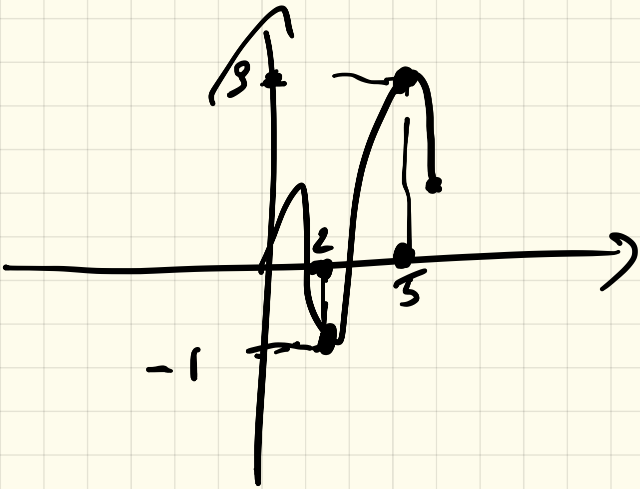
che $f(a) \leq f(x)\ \forall x \in A$

$\Longrightarrow$ $\underline{a}$ si dice punto di min.

$\underline{f(a)}$ si dice valore di min.

IDEM per punto di max

e valore di max



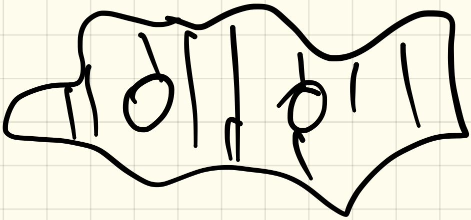
$\text{Min} f = -1$

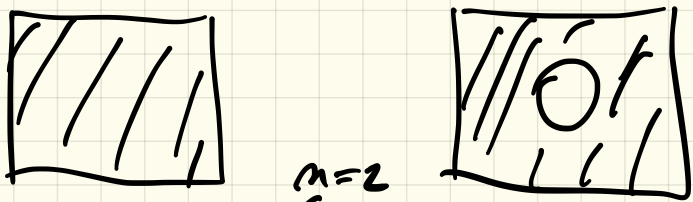
Punto di min. 2

$\text{Max} f = 3$

Punto di max 3

Teo. di Weierstrass





$$f : \Omega \subseteq \mathbb{R}^{n} \longrightarrow \mathbb{R} \qquad (n=2)$$

- f <u>continua</u> (nozione già vista)

- Ω chiuso e limitato

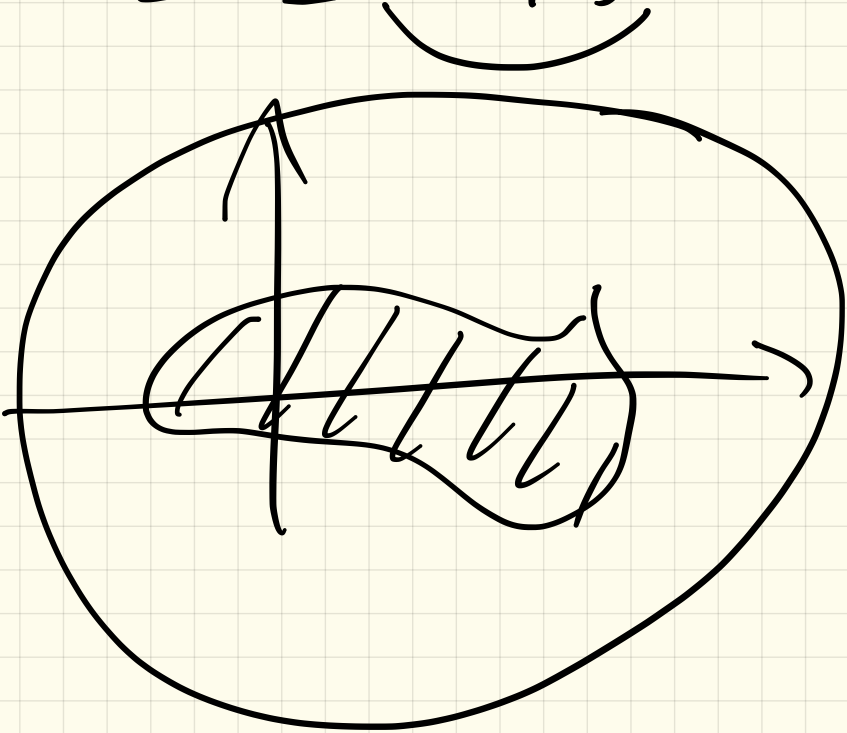
in più variabili? limitato!

Def. $\Omega \subseteq \mathbb{R}^n$ (per semplicità $n=2$)

si dice *limitato* se

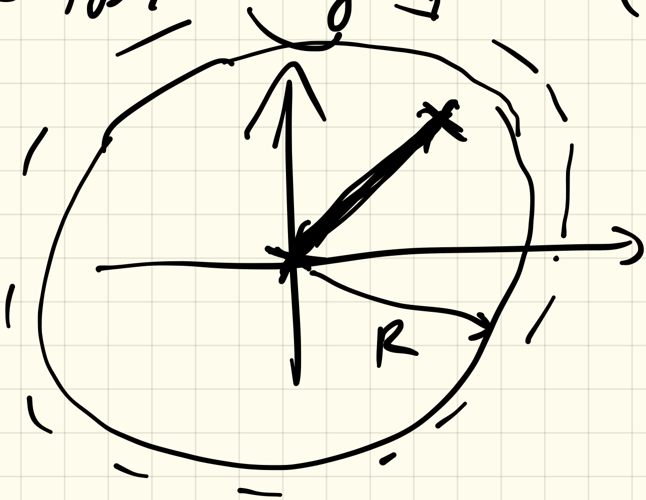
$\exists\, R>0$ t.c.

$$\Omega \subseteq B^n(0,R)$$

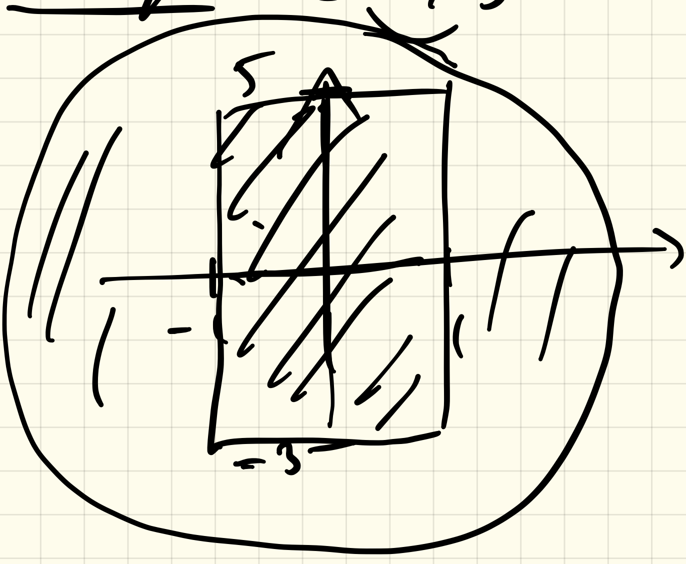


Exemplu (limitată)

$\Omega = \{(x,y) \mid x=y,\ x\in(0,1)\}$

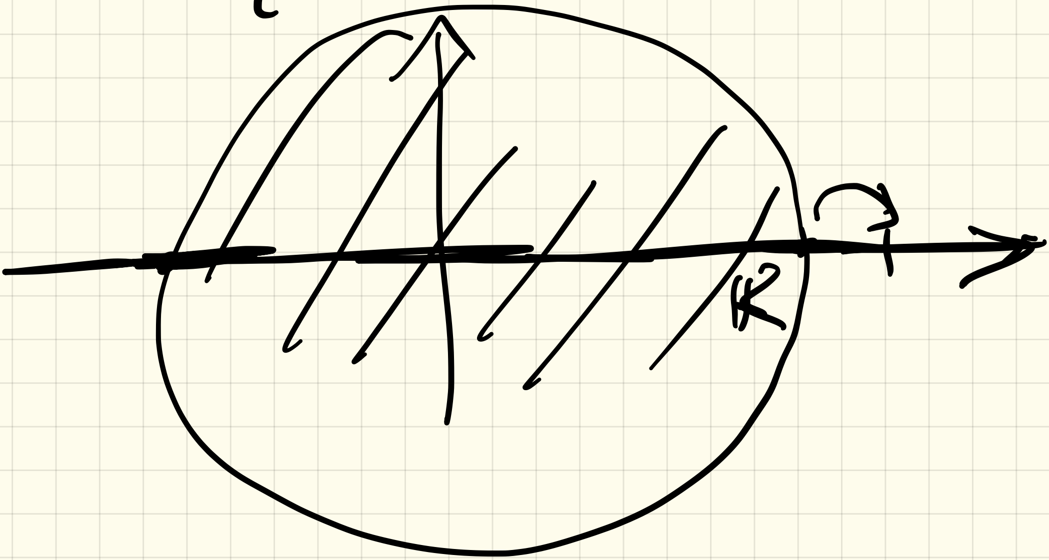


Exemplu $[-1,1]\times[-3,5]$ (limitată)



Esempio (Illimitato)

$$\Omega = \{(x,y) \mid y = 0\}$$

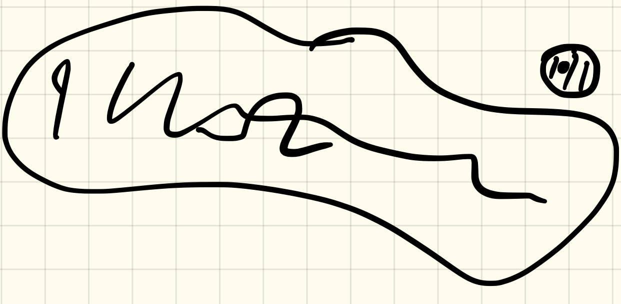


Ω chiuso

Def. $\Omega \subseteq \mathbb{R}^n$ si dice **chiuso**

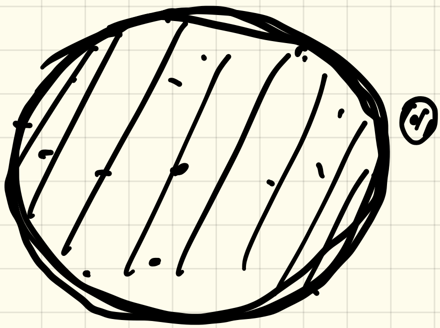
se $\forall\, x \in \mathbb{R}^n \setminus \Omega$

$\exists\, \delta > 0$ t.c. $B^n(x, \delta) \cap \Omega = \emptyset$.



Esempio (di chiusi e non chiusi)

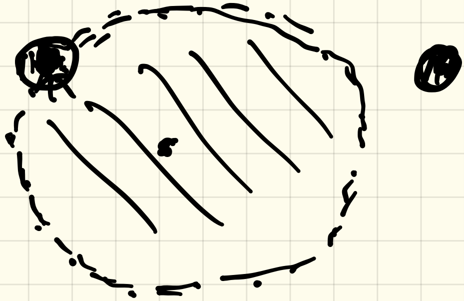
$$\{(x,y) \mid x^2+y^2 \leq 1\} = \Omega$$



Ω è chiuso

Esempio

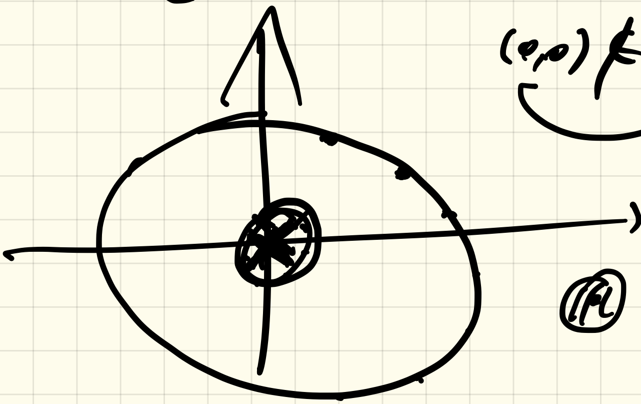
$$\{(x,y) \mid x^2+y^2 < 1\} = \Omega$$



NON É CHIUSO

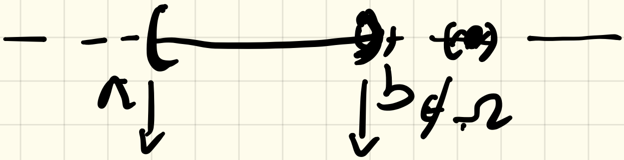
Esempio $\Omega = \{ (x,y) \mid 0 < x^2 + y^2 \leq 1 \}$

$(0,0) \in \bar{\Omega}$



$\boxed{\text{NON É CHIUSO}}$

$\Omega = (a, b]$



Teo di Weierstrass

$f : \Omega \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$

- f continua su Ω
- Ω insieme chiuso e limitato

$\Longrightarrow \exists$ max e min di f su Ω.

$f(\beta) \geq f(x) \ \forall x \in \Omega$

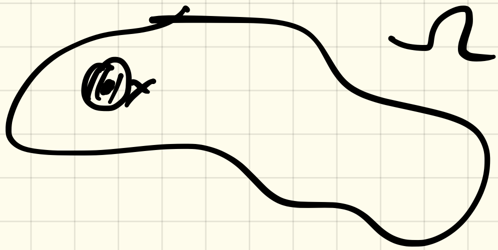
$\exists\, \alpha, \beta \in \Omega$ t.c. $f(\alpha) \leq f(x) \ \forall x \in \Omega$

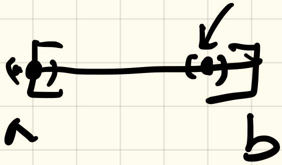
COME TROVARE MAX E MIN OPERATIVAMENTE IN $\mathbb{R}^n$?

$\exists$ Weierstrass $\rightsquigarrow$
- insieme chiuso
- " limitato

<u>Def.</u> $\Omega \subseteq \mathbb{R}^n$ definiamo parte interna di Ω

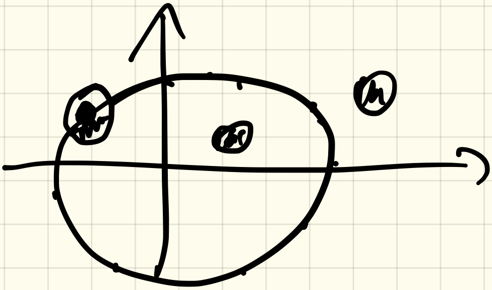
$$\mathring{\Omega} = \left\{ x \in \Omega \mid \exists \delta > 0 \text{ con } B^n(x,\delta) \subset \Omega \right\}$$





$$\mathring{[a,b]} = (a,b)$$

$$\underline{\Omega} = \{(x,y) \mid x^2+y^2 \leq 1\}$$

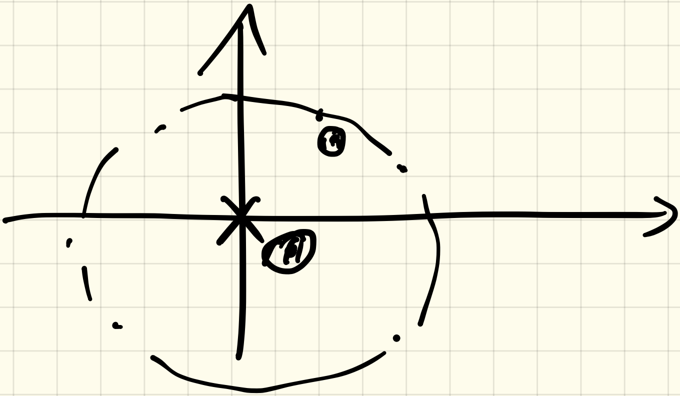


$\mathring{\Omega}$?

$$\underline{\mathring{\Omega} = \{(x,y) \mid x^2+y^2 < 1\}}$$

Esempio

$$\Omega = \{ (x,y) \mid 0 < x^2+y^2 < 1 \}$$



$$\overset{\circ}{\Omega} = \Omega$$

Ω non è chiuso

COME TROVARE MAX E MIN CONCRETAMENTE IN PIÙ VARIABILI.

- Si cercano i punti di $\underbrace{\mathring{\Omega}}$ tali che $\nabla f(x,y) = (0,0) \in \mathbb{R}^2$

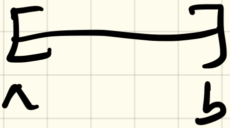
$$\nabla f(x,y,z) = (0,0,0)$$

$$\boxed{\{x_1, \dots, x_n\} \in \mathring{\Omega}}$$

- Si calcola $\underset{\underline{\Omega \setminus \mathring{\Omega}}}{\operatorname{Max}} f$ e $\underset{\underline{\Omega \setminus \mathring{\Omega}}}{\operatorname{Min}} f$

- $\underset{\Omega}{\operatorname{Max}} f = \operatorname{Max}\left\{ \underset{\Omega \setminus \mathring{\Omega}}{\operatorname{Max}} f, f(x_1), \dots, f(x_n) \right\}$

  $\underset{\Omega}{\operatorname{Min}} f = \operatorname{Min}\left\{ \underset{\Omega \setminus \mathring{\Omega}}{\operatorname{Min}} f, f(x_1), \dots, f(x_n) \right\}$



in $(a,b) \rightsquigarrow f' = 0$

$[a,b] \setminus (a,b) = \{a,b\}$

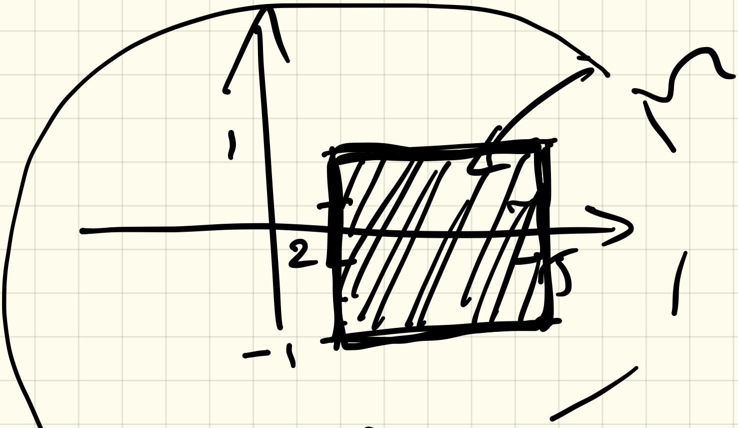
$f(a), f(b)$

Esempio

$\operatorname{Max}_{\Omega} f$ e $\operatorname{Min}_{\Omega} f$

$f(x,y) = x^2 + y^2$

$\Omega = [2,5] \times [-1,1]$



Ω è chiuso $\rightsquigarrow$

Ω è limitato

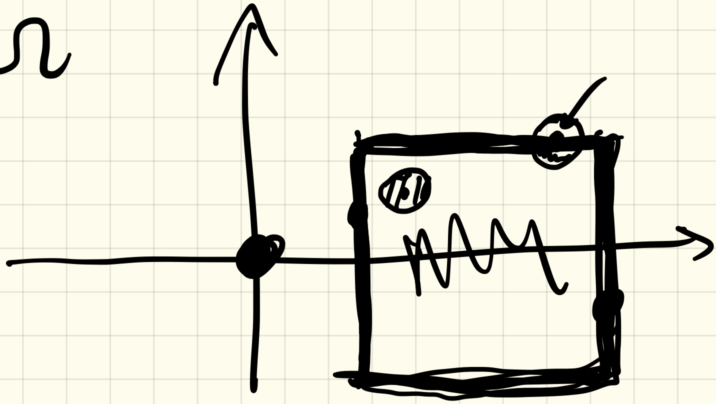
$f(x,y) = x^2+y^2$ è cont.

$$\Downarrow$$

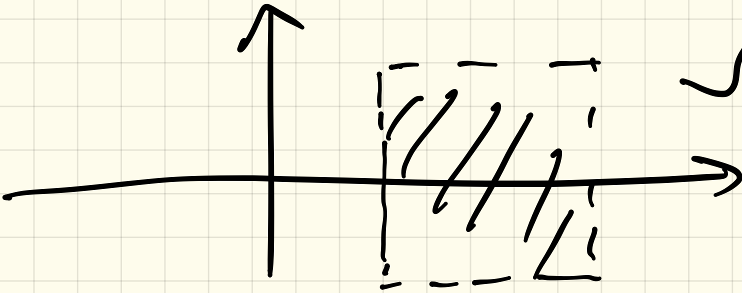
$$\exists \ \max_{\Omega} f \ \text{e} \ \min_{\Omega} f$$

$$\nabla f(x,y) = (0,0) \ \text{in} \ \underline{\mathring{\Omega}}.$$

Ω



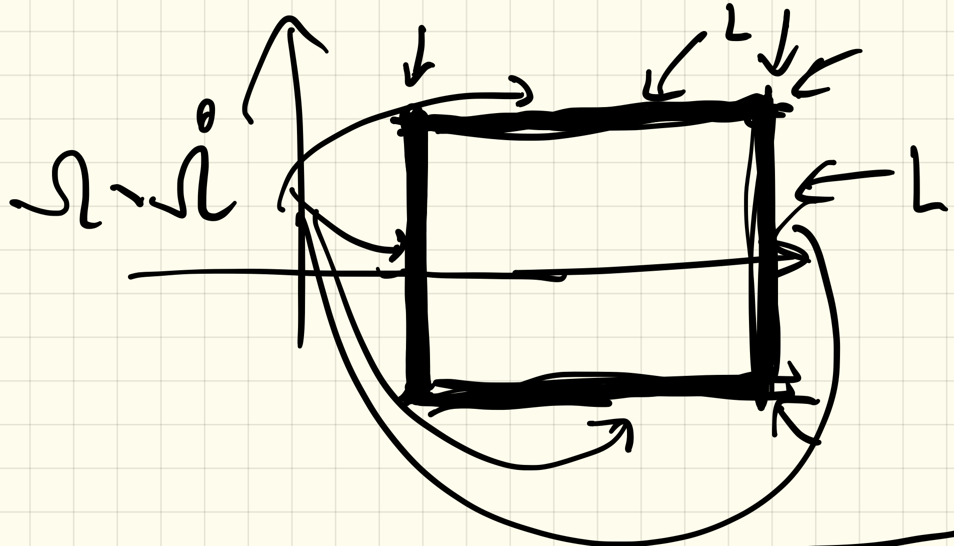
$\mathring{\Omega}$



$$\nabla f(x,y) = (2x, 2y)$$

$$\begin{cases} 2x = 0 \\ 2y = 0 \end{cases} \Longrightarrow x = y = 0$$

$$\boxed{(0,0)} \notin \mathring{\Omega}$$



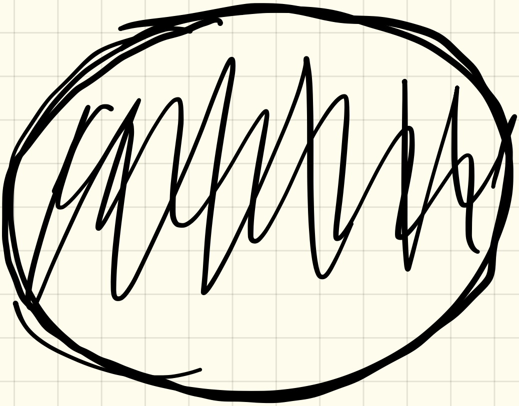
Come studiare

$$\max_{\Omega \setminus \mathring{\Omega}} f \quad e \quad \min_{\Omega \setminus \mathring{\Omega}} f \ ?$$

In 1 variabile è facile:

$$[a,b] \setminus \mathring{[a,b]} = [a,b] \setminus (a,b)$$
$$= \{\underline{a,b}\}$$

$$\Omega = \{x^2+y^2 \le 1\}$$

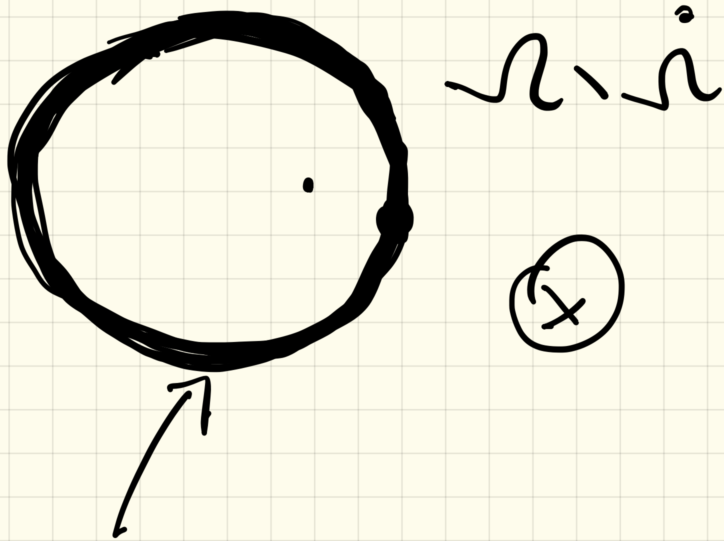


$$\mathring{\Omega} = \{x^2+y^2 < 1\}$$

$$\Downarrow$$

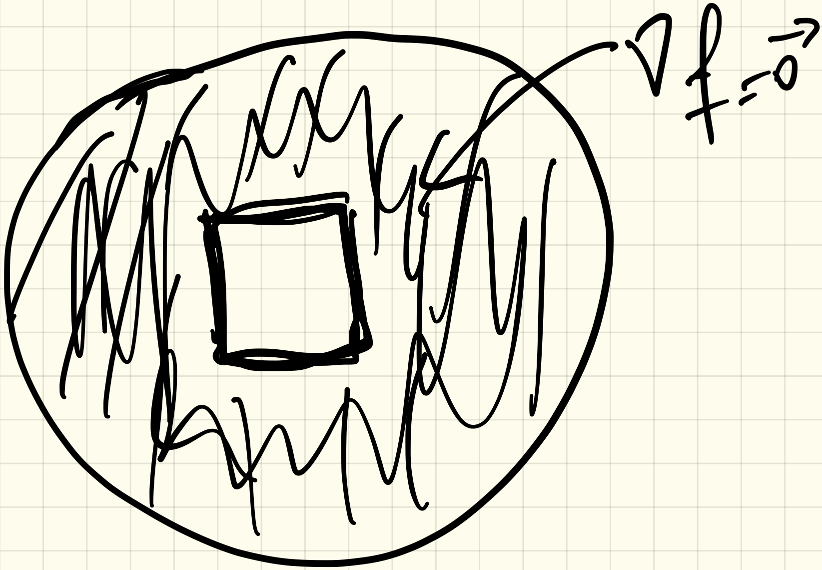
$$\boxed{\nabla f(x,y) = (0,0)}$$

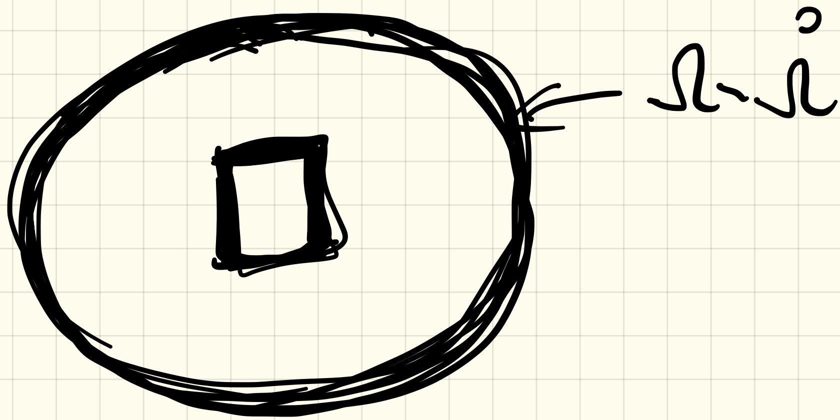
Come studiamo Max e min su $\Omega \setminus \mathring{\Omega}$?



ha infiniti punti e non solo due punti come succede in 1 variabile!!!!

- Parametrizzazione!
- Moltiplicatori di Lagrange!


$\nabla f = \vec{0}$


$\Omega - \mathring{\Omega}$