Analisi Matematica - Prova scritta parziale n.3 (7.4.09)

Soluzioni [1]

1.

- Per $x \to 0^+$ $\quad f(x) = e^{\lg x \, \lg(1+\operatorname{tg} x)} \sim e^{x \lg x} \to 1$

  per $x \to 0^-$ $\quad f(x) = \dfrac{\lg(1+x+x^2)}{x} \sim \dfrac{x}{x} \to 1$

  Si elimina la discontinuità ponendo $f(0)=1$

- $$f'(x) = \begin{cases} (1+\operatorname{tg} x)^{\lg x} \left(\dfrac{\lg(1+\operatorname{tg} x)}{x} + \dfrac{\lg x}{(1+\operatorname{tg} x)\cos^2 x} \right) & \text{per } 0 < x < \frac{\pi}{2} \\ \dfrac{x(1+2x) - (1+x+x^2)\lg(1+x+x^2)}{x^2(1+x+x^2)} & \text{per } -1 < x < 0. \end{cases}$$

- per $x \to 0^+$ $\quad f'(x) \sim \dfrac{x}{x} + \dfrac{\lg x}{} \to -\infty$

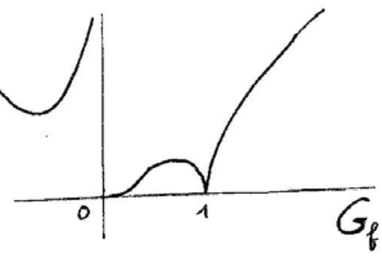
  per $x \to 0^-$ $\quad f'(x) \sim \dfrac{x(1+2x) - (1+x+x^2)\lg(1+x+x^2)}{x^2} \overset{H}{=}$

  $$\dfrac{2x - (1+2x)\lg(1+x+x^2)}{2x} \sim \dfrac{x}{2x} \to \dfrac{1}{2}$$

  Dunque, $f'(0)$ non esiste ; $x=0$ punto misto.

2.

$$f(x) = \sqrt{|x-1|}\; e^{-1/x}$$



C.E. $\quad \mathbb{R} - \{0\}$

SGN $\quad$ positiva; nulla per $x=1$

LIM $\quad$ per $x \to 0^+$ $\; f(x) \to 0$ $\;$ D.E. da destra

per $x \to 0^-$ $\; f(x) \to +\infty$ $\;$ asintoto verticale

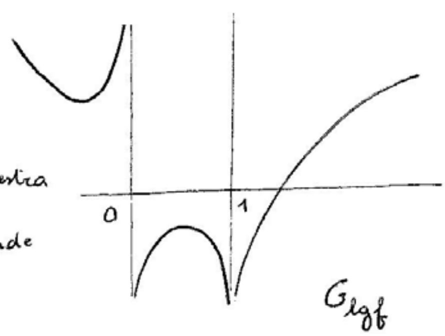
per $x \to \pm\infty$ $\; f(x) \sim \sqrt{|x|} \to +\infty$ $\;$ senza asintoto

DRV $\quad f'(x) = \dfrac{x^2+2x-2}{2x^2\sqrt{|x-1|}}\, e^{-1/x}\, \operatorname{sgn}(x-1)$

$-1-\sqrt{3} \qquad 0 \qquad -1+\sqrt{3} \qquad 1$

per $x \to 0^+$ $\quad f'(x) \sim \dfrac{e^{-1/x}}{x^2} \to 0$ $\quad$ tg. orizz. a destra

per $x \to 1^\pm$ $\quad f'(x) \sim \dfrac{\operatorname{sgn}(x-1)}{2e\sqrt{|x-1|}} \to \pm\infty$ $\quad$ cuspide



3.

Sviluppiamo al secondo ordine:

$$\lg(1+\operatorname{sen} x) - x\cos^2 x = \lg\left(1+x+o(x^2)\right) - x\left(1-\frac{1}{2}x^2+o(x^2)\right)^2$$
$$= x - \frac{1}{2}x^2 - x + o(x^2) = -\frac{1}{2}x^2$$

$$\sqrt{1+x} = 1+\frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$$

$$\sqrt[4]{1+x} = 1+\frac{1}{4}x - \frac{3}{32}x^2 + o(x^2)$$

$$\sqrt[4]{1+2x} = 1+\frac{1}{2}x - \frac{3}{8}x^2 + o(x^2)$$

$$\sqrt{1+x} - \sqrt[4]{1+2x} = \frac{1}{4}x^2 + o(x^2)$$

Il limite vale -2.

Soluzioni [2]

1.

. Per $x \to 1^-$ $\quad f(x) = \dfrac{\lg x}{\operatorname{sen}(x-1)} \sim \dfrac{x-1}{x-1} \to 1$

per $x \to 1^+$ $\quad f(x) = e^{\lg x\,(\lg 3 - \lg(x-1))} \sim e^{-(x-1)\lg(x-1)} \to 1$

Si elimina la discontinuità ponendo $f(1) = 1$.

$$. \quad f'(x) = \begin{cases} \dfrac{\dfrac{\operatorname{sen}(x-1)}{x} - \lg x \cos(x-1)}{\operatorname{sen}^2(x-1)} & \text{per } 0 < x < 1 \\[2ex] \left(\dfrac{3}{x-1}\right)^{\lg x} \left\{ \dfrac{1}{x}\left(\lg 3 - \lg(x-1)\right) - \dfrac{\lg x}{x-1} \right\} & \text{per } x > 1 \end{cases}$$

. Per $x \to 1^-$ $\quad f'(x) \sim \dfrac{\operatorname{sen}(x-1) - x \lg x \cos(x-1)}{(x-1)^2} \underset{H.}{=}$

$$\underset{H.}{=} \frac{-\lg x \cos(x-1) + x \lg x \operatorname{sen}(x-1)}{2(x-1)}$$

$$\sim \frac{-(x-1)}{2(x-1)} \to -\frac{1}{2}$$

per $x \to 1^+$ $\quad f'(x) \sim -\lg(x-1) - \dfrac{(x-1)}{(x-1)} \to +\infty$

Dunque, $f'(1)$ non esiste; $x = 1$ punto misto.

2. C.E. $\quad \mathbb{R} - \{0\}$

SGN $\quad$ positiva; nulla per $x = 1$

LIM $\quad$ per $x \to 0^+$ $\quad f(x) \to 0$ $\quad$ D.E. da destra

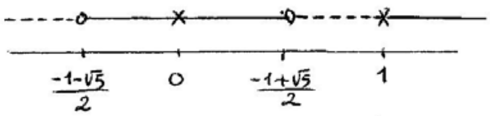
per $x \to 0^-$ $\quad f(x) \to +\infty$ $\quad$ asintoto verticale

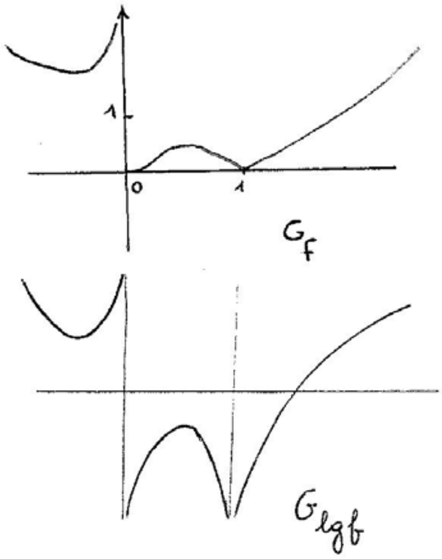
per $x \to \pm\infty$ $\quad f(x) \to +\infty$

$y = x - 2$ $\quad$ asintoto obliquo per $x \to +\infty$

$y = 2 - x$ $\quad$ asintoto obliquo per $x \to -\infty$

DRV $$f'(x) = \operatorname{sgn}(1-x)\, e^{-1/x}\, \frac{-x^2-x+1}{x^2} =$$
$$= \operatorname{sgn}(x-1)\, e^{-1/x}\, \frac{x^2+x-1}{x^2}.$$





per $x \to 0^+$ $\quad f'(x) \simeq \frac{1}{x^2} e^{-\frac{1}{x}} \to 0$

tg. orizzontale destra

per $x \to 1^\pm$ $\quad f'(x) \sim \frac{\operatorname{sgn}(x-1)}{e} \to \pm \frac{1}{e}$

punto angoloso

3. Sviluppiamo il numeratore al 4° ordine:

$$2\lg\left(1 - \tfrac{1}{2}x^2 + \tfrac{1}{24}x^4 + o(x^4)\right) + x\left(x - \tfrac{x^3}{6} + o(x^4)\right) =$$
$$= 2\left[\left(-\tfrac{1}{2}x^2 + \tfrac{1}{24}x^4\right) - \tfrac{1}{2}\left(\tfrac{1}{4}x^4\right)\right] + x^2 - \tfrac{x^4}{6} + o(x^4) =$$
$$= -\tfrac{1}{3}x^4 + o(x^4).$$

Sviluppiamo il denominatore al secondo ordine:

$$\sqrt[3]{1+x} = 1 + \tfrac{1}{3}x - \tfrac{1}{9}x^2 + o(x^2)$$
$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} = 1 - \tfrac{1}{2}x + \tfrac{3}{8}x^2 + o(x^2)$$
$$\frac{1}{\sqrt{1-\frac{2}{3}x}} = 1 + \tfrac{1}{3}x + \tfrac{1}{6}x^2 + o(x^2)$$

Il denominatore si approssima con $-\frac{5}{18}x^2$.

Il limite vale 0.