Ist. Mat. I - C 1A
14/12/23

$\boxed{18/7/23}$

$$f(x) = \frac{2x^2+x}{x^2-1}$$

(A) D $\quad \mathbb{R} \setminus \{\pm 1\} = (-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

(B) zeri $\quad x(2x+1) \qquad x=0 \qquad x=-1/2$

(C) limiti agli estremi

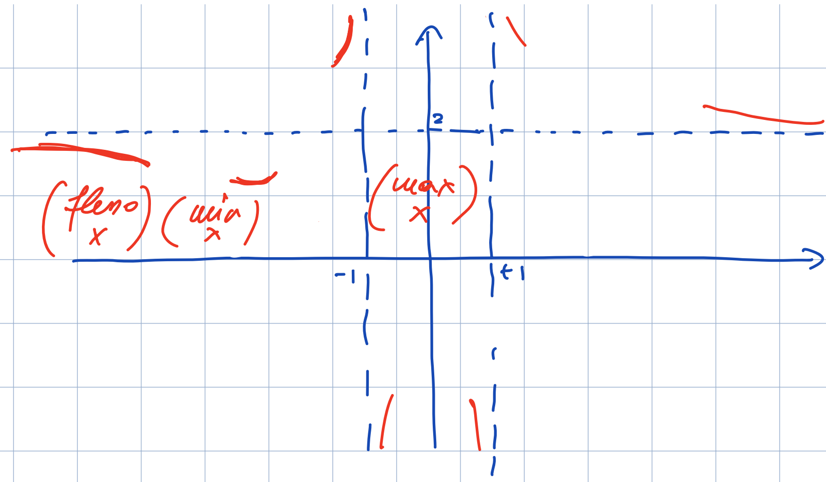
$$\frac{2x^2+x}{(x+1)(x-1)}$$

$$\lim_{\pm\infty} = 2 \ ;$$

$$\frac{2x^2+x}{x^2-1} - 2 = \frac{2x^2+x-2x^2+2}{x^2-1} = \frac{x+2}{x^2-1} \begin{array}{l} \nearrow 0^+ \text{ in } +\infty \\ \searrow 0^- \text{ in } -\infty \end{array}$$

$$\lim_{(-1)^\pm} = \frac{2-1}{0^\pm \cdot (-2)} = \mp\infty$$

$$\lim_{(+1)^\pm} = \frac{2+1}{2 \cdot 0^\pm} = \pm\infty$$



(D) max/min rel.

$$f'(x) = \left(\frac{2x^2+x}{x^2-1}\right)' = \frac{(4x+1)(x^2-1)-2x(2x^2+x)}{(x^2-1)^2}$$

$$= \frac{\cancel{4x^3}-4x+x^2-1\cancel{-4x^3}-2x^2}{(x^2-1)^2} = -\frac{x^2+4x+1}{(x^2-1)^2}$$

$-2\pm\sqrt{3}$

$-2-\sqrt{3}<-1$ min rel

$-1<-2+\sqrt{3}<0$ max rel

(E) Provare che f ha flessi

$$f''(x) = \left(-\frac{x^2+4x+1}{(x^2-1)^2}\right)'$$

$$= -\frac{(2x+4)(x^2-1)^{\cancel{2}}-2\cancel{(x^2-1)}2x\cdot(x^2+4x+1)}{(x^2-1)^{\cancel{4}\,3}}$$

$$= -\frac{2x^3 - 2x + 4x^2 - 4 - 4x^3 - 16x^2 - 4x}{(x^2-1)^3}$$

$$= \frac{2x^3 + \dots}{(x^2-1)^3}$$ ha almeno uno zero

12/9/23

$$f(x) = \frac{3x^2 - (x+1)\cdot|x|}{x-1} = \begin{cases} \dfrac{3x^2 - (x+1)\cdot x}{x-1} = \dfrac{2x^2 - x}{x-1} & x \geq 0 \\[2ex] \dfrac{3x^2 + (x+1)\cdot x}{x-1} = \boxed{\dfrac{4x^2 + x}{x-1}} & x > 0 \end{cases}$$

(A) D $\qquad D = \mathbb{R} \setminus \{1\}$

(B) limiti agli estremi

$$\lim_{-\infty} = -\infty, \quad \lim_{1^\pm} = \frac{2-1}{0^\pm} = \pm\infty, \quad \lim_{+\infty} = +\infty$$

(C) asintoti verticale $x = 1$

$$\lim_{-\infty} \frac{f(x)}{x} = 4 = m? \qquad q = \lim\left(f(x) - 4x\right)$$

$$= \lim\left(\frac{4x^2+x}{x-1} - 4x\right) = \lim\left(\frac{4x^2+x-4x^2+4x}{x-1}\right)$$

$$= \lim \frac{5x}{x-1} = 5$$

obl. sin. $y = 4x + 5$

sopra o sotto?

$$\frac{4x^2+x}{x-1} - (4x+5) = \frac{\cancel{4x^2}+\cancel{x}-\cancel{4x^2}-\cancel{5x}+\cancel{4x}+5}{x-1} \to 0^-$$

sotto

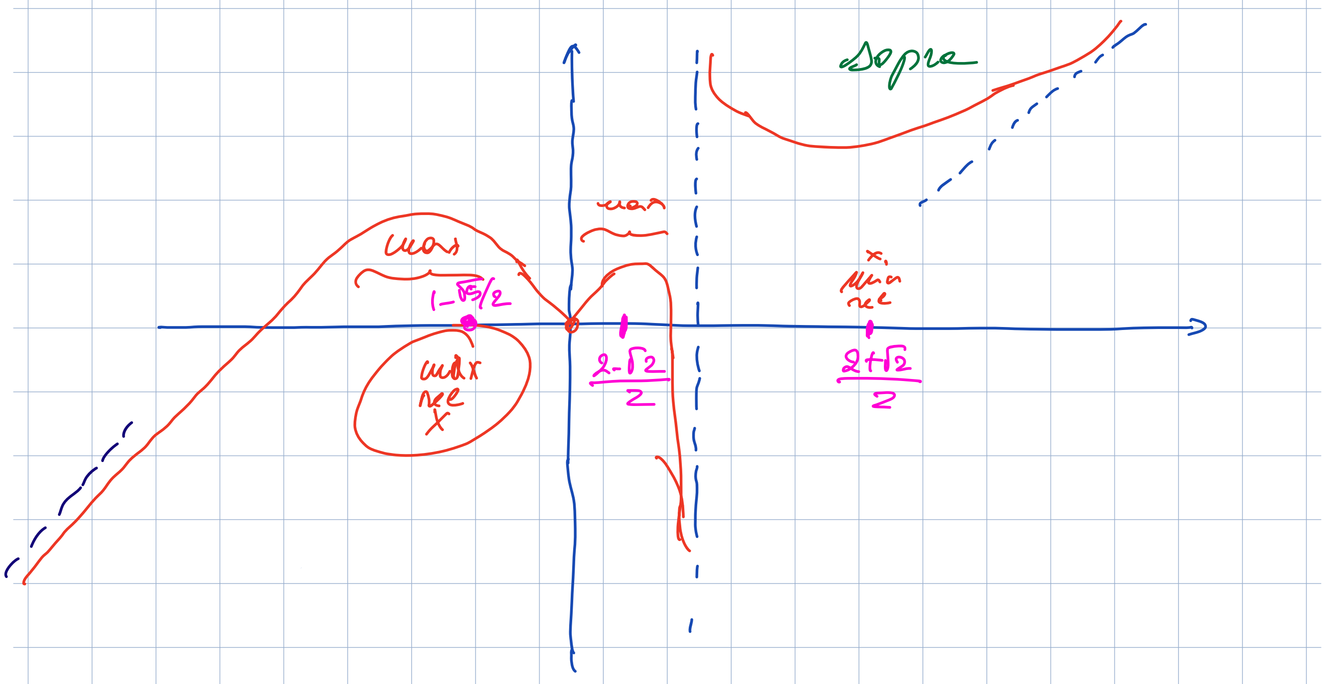
$$\lim_{+\infty} \frac{f(x)}{x} = 2 = m \qquad q = \lim\left(\frac{2x^2-x}{x-1} - 2x\right)$$

$$= \lim \frac{2x^2-x-2x^2+2x}{x-1} = \lim \frac{x}{x-1} = 1$$

obl. dx $2x+1$

sopra o sotto?

$$\frac{2x^2-x}{x-1} - (2x+1) = \frac{\cancel{2x^2}-\cancel{x}-\cancel{2x^2}-\cancel{x}+\cancel{2x}+2}{x-1} \to 0^+$$



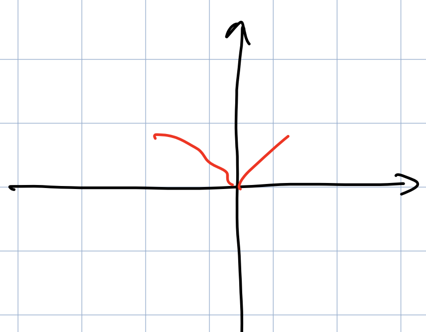
(D) Trovare f' dove esiste o $f'_\pm$.

$x>0$
$$\left(\frac{2x^2-x}{x-1}\right)' = \frac{(4x-1)(x-1)-(2x^2-x)}{(x-1)^2}$$
$$= \frac{4x^2-4x-x+1-2x^2+x}{(x-1)^2} = \frac{2x^2-4x+1}{(x-1)^2}$$
$\longrightarrow +1$ in 0^+

$x<0$
$$\left(\frac{4x^2+x}{x-1}\right)' = \frac{(8x+1)(x-1)-(4x^2+x)}{(x-1)^2}$$
$$= \frac{8x^2-8x+x-1-4x^2-x}{(x-1)^2} = \frac{4x^2-8x-1}{(x-1)^2}$$
$\longrightarrow -1$ in 0^-

$f'(x)$ esiste $\forall x \in D$, $x \neq 0$.
$f'_\pm(0) = \pm 1$ 0 pto angoloso

(E) max/min loc.



0 min loc.

$x>0 \qquad f'(x) = \dfrac{2x^2-4x+1}{(x-1)^2} \qquad \dfrac{2\pm\sqrt{4-2}}{2} = \dfrac{2\pm\sqrt{2}}{2}$

$$0 < \underbrace{\frac{2-\sqrt{2}}{2}}_{max} < 1 < \underbrace{\frac{2+\sqrt{2}}{2}}_{min}$$

$$\begin{array}{c|ccccccccc} & 0 & & \frac{2-\sqrt{2}}{2} & & 1 & & \frac{2+\sqrt{2}}{2} & \\ \hline f' & & + & 0 & - & \times & - & 0 & + \\ & & \nearrow & & \searrow & & \searrow & & \nearrow \end{array}$$

$x<0 \qquad f'(x) = \dfrac{4x^2-8x-1}{(x-1)^2}$

$$\frac{4\pm\sqrt{16+4}}{4} = 1 \pm \frac{\sqrt{5}}{2} \to \cancel{1+\frac{\sqrt{5}}{2}}, \quad 1-\frac{\sqrt{5}}{2} < 0$$

I prova – 25/11/22

$$f(x) = \frac{x\cdot(x-|2x-8|)}{x+1}$$

$$f(x) = \begin{cases} \dfrac{x\cdot(x-(2x-8))}{x+1} = \dfrac{8x-x^2}{x+1} & x \geq 4 \\[2ex] \dfrac{x\cdot(x+(2x-8))}{x+1} = \dfrac{3x^2-8x}{x+1} & x<4 \end{cases}$$

(C) (nulla $x=8$, $x=0$, $x=8/3$)

(A) D $\mathbb{R} \setminus \{-1\}$

(B) Asintoti:

$$\lim_{-\infty} f = -\infty \qquad \lim_{-\infty} \frac{f(x)}{x} = 3 = m$$

$$\lim_{-\infty} \left(f(x) - 3x\right) = \lim_{-\infty} \left(\frac{3x^2-8x}{x+1} - 3x\right)$$

$$= \lim_{-\infty} \left(\frac{3x^2-8x-3x^2-3x}{x+1}\right) = -11 = q$$

obl. sin. $y = 3x - 11$

$$\frac{3x^2-8x}{x+1} - (3x-11) = \frac{\cancel{3x^2} \cancel{-8x} \cancel{-3x^2} \cancel{+11x} \cancel{-3x} + 33}{x+1} \longrightarrow 0^-$$

sotto

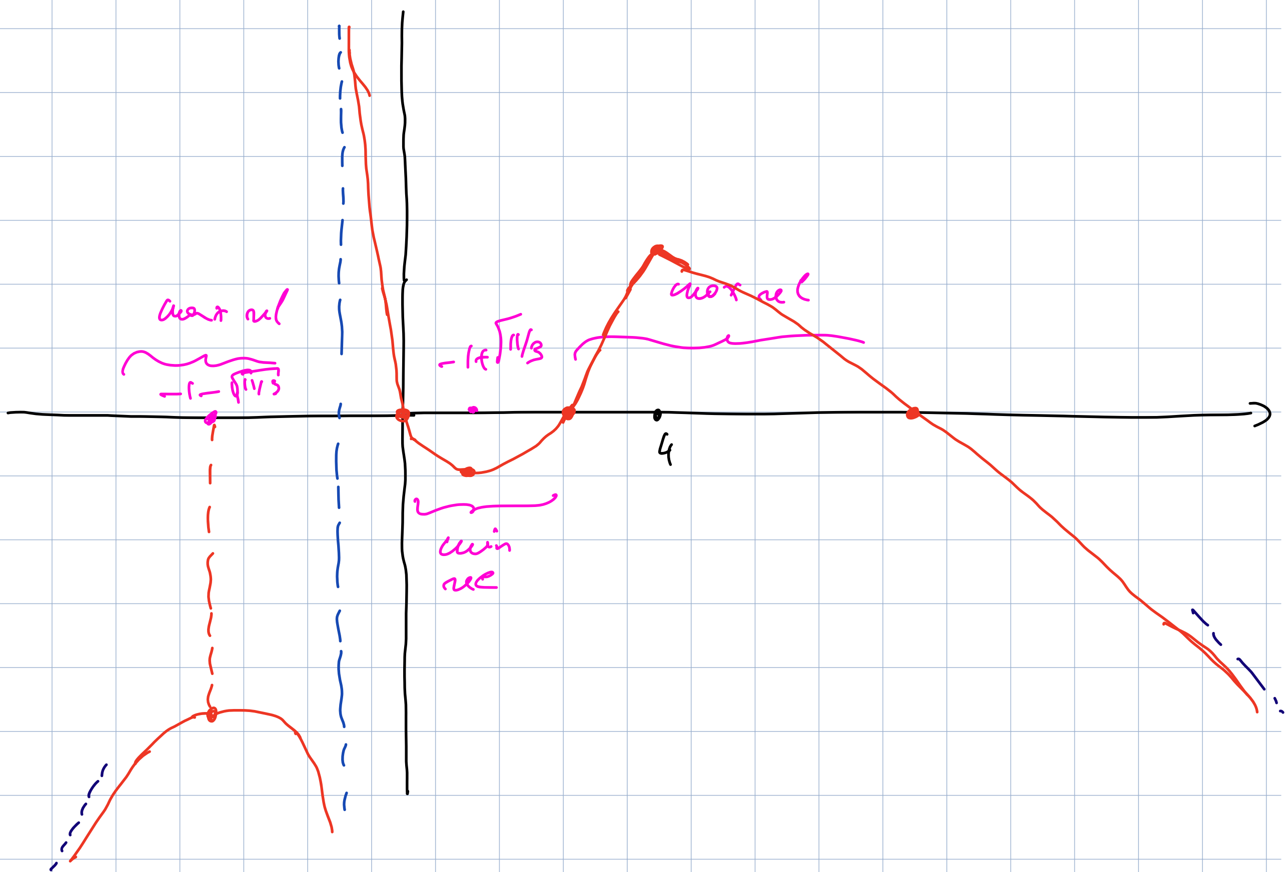
$$\lim_{+\infty} f = -\infty \qquad \lim_{+\infty} \frac{f(x)}{x} = -1 = m$$

$$q = \lim_{+\infty} \left(\frac{8x - x^2}{x+1} + x\right) = \lim_{+\infty} \frac{8x - x^2 + x^2 + x}{x-1} = 9$$

obl. dx $y = -x + 9$

$$\frac{8x-x^2}{x+1} - (-x+9) = \frac{\cancel{8x} \cancel{-x^2} \cancel{+x^2} \cancel{+x} \cancel{-9x} - 9}{x+1} \longrightarrow 0^-$$

sotto



$$\lim_{x\to(-1)^\pm} \frac{3x^2-8x}{x+1} = \frac{3+8}{0^\pm} = \pm\infty$$

(D) max/min rel.

$x>4 \qquad f'(x) = \left(\frac{8x-x^2}{x+1}\right)' = \frac{(8-2x)(x+1)-(8x-x^2)}{(x+1)^2}$

$$= \frac{\cancel{8x}+8-2x^2-2x\cancel{-8x}+x^2}{(x+1)^2} = -\frac{x^2+2x-8}{(x+1)^2}$$

$$= -\frac{(x+4)(x-2)}{(x+1)^2} \qquad \text{sempre negativa}$$

$$f'_+(4) = -\frac{16}{25}$$

$x > 4 \qquad f'(x) = \left(\frac{3x^2-8x}{x+1}\right)' = \frac{(6x-8)(x+1)-(3x^2-8x)}{(x+1)^2}$

$$= \frac{\cancel{6x^2}+6x\cancel{-8x}-8\cancel{-3x^2}\cancel{+8x}}{(x+1)^2} = \frac{3x^2+6x-8}{(x+1)^2}$$

$$\frac{-3 \pm \sqrt{9+24}}{3} = -1 \pm \sqrt{\frac{11}{3}} \quad \begin{cases} \sim -3 \\ \sim +1 \end{cases}$$

$$f'_-(4) = \frac{3 \cdot 16 + 6 \cdot 4 - 8}{25} > 0$$

II di prova - 6/12/22

$$f(x) = x \cdot e^{\frac{1}{x^2}}$$

(A) D $\qquad D = \mathbb{R} \setminus \{0\}$

(B) asintoti

$$\lim_{\pm\infty} f = \pm\infty \qquad \lim_{\pm\infty} \frac{f(x)}{x} = \lim_{\pm\infty} e^{\frac{1}{x^2}} = 1 \; \exists \, m$$

$$q \; \exists \; \lim_{\pm\infty} \underbrace{(f(x) - x)}_{\infty - \infty} = \lim_{\pm\infty} x \cdot (e^{\frac{1}{x^2}} - 1) \qquad \infty \cdot 0$$

$$= \lim_{\pm\infty} x \cdot \left(\left(1 + \frac{1}{x^2} + o\left(\frac{1}{x^2}\right)\right) - 1\right)$$

$$= \lim_{\pm\infty} \left(\frac{1}{x} + o\left(\frac{1}{x}\right)\right) = 0$$

$y = x$ as. obl. sin/dx

$$\frac{e^{\frac{1}{x^2}}-1}{\frac{1}{x}} \qquad \frac{0}{0}$$

$$\downarrow$$

$$\frac{e^{\frac{1}{x^2}}\cdot\left(-\frac{2}{x^3}\right)}{-\frac{1}{x^2}} = \frac{2e^{\frac{1}{x^2}}}{x}$$

$$g(x) = x\cdot e^{\frac{1}{\sqrt{x}}} \qquad \text{su } (0,+\infty)$$

$$\lim_{+\infty} g = +\infty\cdot 1 = +\infty$$

$$\lim_{+\infty}\frac{g(x)}{x} = 1 \overset{?}{=} m$$

$$q \overset{?}{=} \lim_{+\infty}\left(g(x)-x\right) = \lim_{+\infty} x\cdot\left(e^{\frac{1}{\sqrt{x}}}-1\right)$$

$$= \lim_{+\infty}\left(x\cdot\left(\left(1+\frac{1}{\sqrt{x}}+o\left(\frac{1}{\sqrt{x}}\right)\right)-1\right)\right)$$

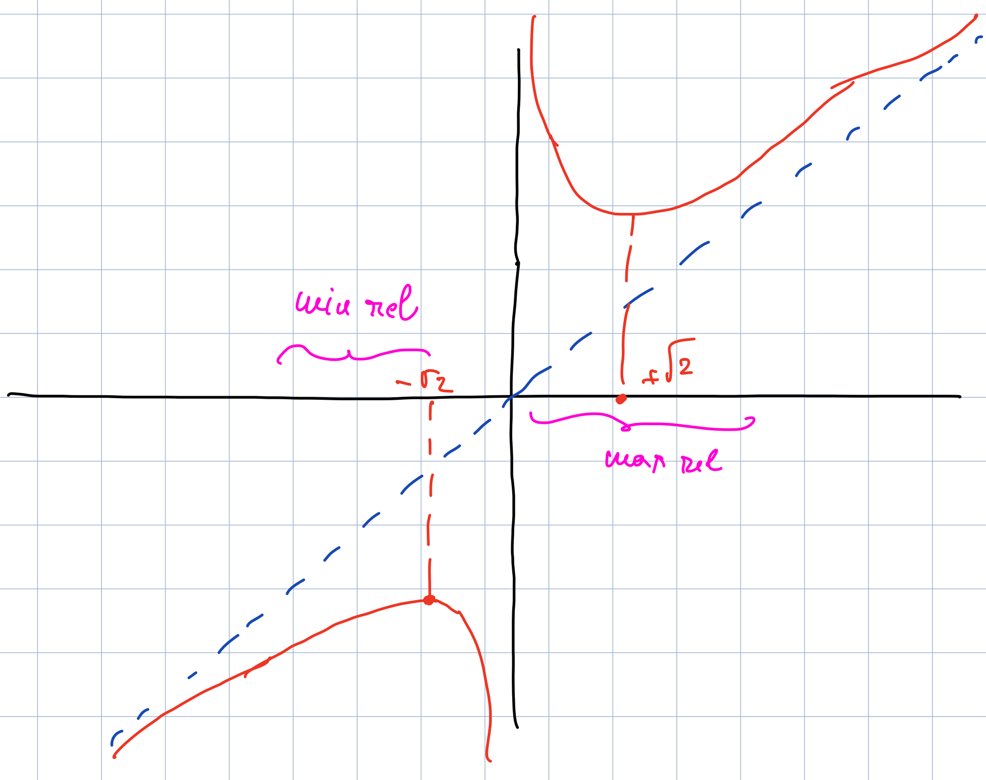
$$= \lim_{+\infty}\left(\sqrt{x}+o(\sqrt{x})\right) = +\infty$$

sopra/sotto $\qquad f(x)-x \to 0^{\pm}$ in $\pm\infty$

sopra a dx $\quad$ sotto a sin.

(oss: f è dispari)

$$\lim_{x\to 0^\pm} f(x) = \lim_{x\to 0^\pm} x \cdot e^{\frac{1}{x^2}} \qquad 0^\pm \cdot (+\infty) \qquad \pm\infty$$



$$f'(x) = \left(x \cdot e^{\frac{1}{x^2}} \right)' = e^{\frac{1}{x^2}} + x \cdot e^{\frac{1}{x^2}} \cdot \left(-\frac{2}{x^3} \right)$$

$$= e^{\frac{1}{x^2}} \cdot \left(1 - \frac{2}{x^2} \right) \qquad \text{nulla in } \pm\sqrt{2}$$

	$-\sqrt{2}$		0		$\sqrt{2}$	
f': $+$	0	$-$	$\times$	$-$	0	$+$
$\nearrow$	max rel	$\searrow$		$\searrow$	min rel	$\nearrow$

$$f''(x) = e^{\frac{1}{x^2}} \cdot \left(-\frac{2}{x^3}\right) \cdot \left(1 - \frac{2}{x^2}\right) + e^{\frac{1}{x^2}} \cdot \left(\frac{4}{x^3}\right)$$

$$= e^{\frac{1}{x^2}} \left(-\frac{2}{x^3} + \frac{4}{x^5} + \frac{4}{x^3}\right)$$

$$= e^{\frac{1}{x^2}} \left(\frac{4}{x^5} + \frac{2}{x^3}\right) = \frac{2e^{1/x^2}(2+x^2)}{x^5}$$

concorde con x