Ist. Mat. I - CIA
13/12/22

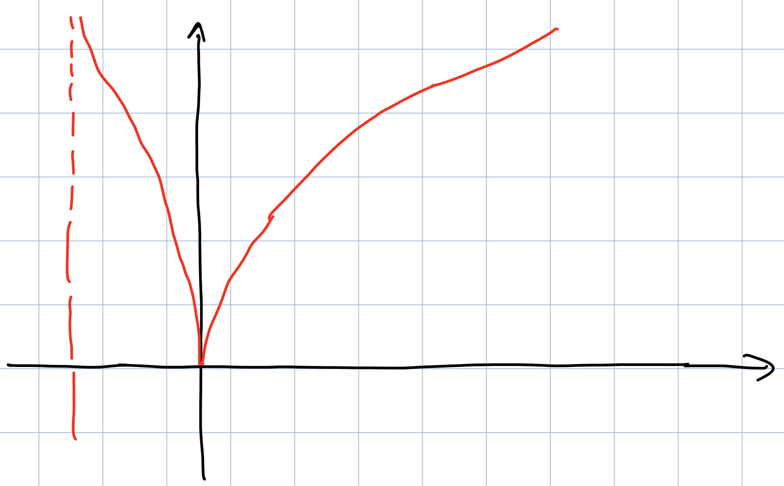
② p. 171 (38) $\qquad f(x) = \log^2\left(1+\sqrt[3]{x}\right)$

$D = (-1, +\infty)$

$\lim_{x \to -1^+} f(x) = +\infty$

Su D esiste $f'(x)$ escluso $x=0$. Vicino a $y=0$ abbiamo $\log(1+y) \cong y$

$\Rightarrow f(x) \cong \sqrt[3]{x^2} = x^{2/3}$ vicino a $x=0$



$$f'(x) = \underbrace{\underbrace{2\log\left(1+\sqrt[3]{x}\right)}_{\downarrow\, 0} \cdot \underbrace{\frac{1}{1+\sqrt[3]{x}}}_{\downarrow\, 1} \cdot \frac{1}{3}\underbrace{x^{-2/3}}_{\downarrow\, \infty}}_{\infty}$$

Oss: molti software richiesti di tracciare grafico di $\log^2(1+\sqrt[3]{x})$ lo fanno solo per $x>0$. Ragione: a^y ($y \in \mathbb{R}$) ha senso solo per $a>0$. Ricordio:

$x>0 \longrightarrow \log^2(1+\sqrt[3]{x})$

$x<0 \longrightarrow \log^2(1-\sqrt[3]{-x})$

Pearson (26) ② Trovare max/min di $\cos(x)+x\cdot\sin(x)$ su $[0,\pi]$

La funzione è continua su $[0,\pi]$ $\Rightarrow$ ha max/min ass.

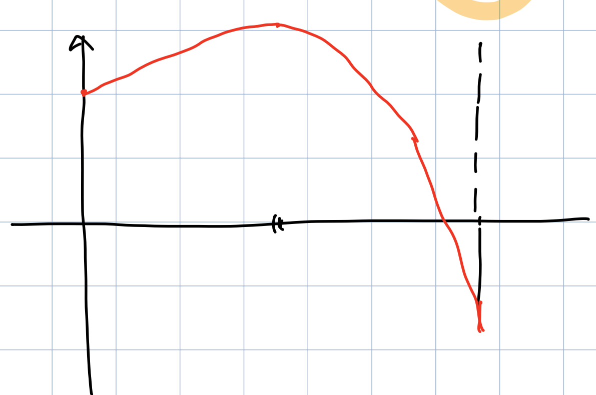
E' derivabile $\Rightarrow$ i max/min rel. sono o gli estremi o i punti x in cui $f'(x)=0$.

$$f'(x) = -\sin(x)+\sin(x)+x\cdot\cos(x) = x\cdot\cos(x)$$

nulla in ($x=0$ e) $x=\frac{\pi}{2}$

(pos. su $(0,\frac{\pi}{2})$, neg. su $(\frac{\pi}{2},\pi]$.)

$f(0)=1 \qquad f(\pi/2) = 0+\frac{\pi}{2} = \frac{\pi}{2} \qquad f(\pi) = -1$



27 (1) $\lim_{x\to 0} \dfrac{x\cdot\cos(x) - \sin(x)}{x^2\cdot\sin(2x)} = \dfrac{0}{0}$

De L'Hôpital (A)

Taylor per funzioni note (B)

"trovare Taylor per num e den."

(B) $\dfrac{x\cdot\left(1-\frac{1}{2}x^2+o(x^3)\right)-\left(x-\frac{1}{6}x^3+o(x^3)\right)}{x^2\cdot\left(2x+o(x^2)\right)}$

$= \dfrac{x-\frac{1}{2}x^3-x+\frac{1}{6}x^3+o(x^3)}{2x^3+o(x^3)} = \dfrac{-\frac{1}{3}x^3+o(x^3)}{2x^3+o(x^3)} = -\dfrac{1}{6}$

(A) $\dfrac{x\cdot\cos(x)-\sin(x)}{x^2\cdot\sin(2x)} = \dfrac{0}{0}$

$\dfrac{\cancel{\cos(x)}-x\cdot\sin(x)-\cancel{\cos(x)}}{2x\cdot\sin(2x)+2x^2\cdot\cos(2x)} = -\dfrac{1}{2}\dfrac{\sin(x)}{\sin(2x)+x\cdot\cos(2x)} = \dfrac{0}{0}$

$-\dfrac{1}{2}\cdot\dfrac{\cos(x)}{2\cos(2x)+\cos(2x)-2x\cdot\sin(2x)} = -\dfrac{1}{2}\cdot\dfrac{1}{2+1+0}$

$= -\dfrac{1}{6}$

$$⑳ ② \quad \lim_{x\to 0} \frac{x\cdot e^{2x^2}-\sinh(x)}{x^2\cdot \log(1+3x)}$$

$$\frac{x\cdot\left(1+2x^2+o(x^3)\right)-\left(x+\frac{1}{6}x^3+o(x^3)\right)}{x^2\cdot\left(3x+o(x)\right)}$$

$$=\frac{\frac{11}{6}x^3+o(x^3)}{3x^3+o(x^3)} \longrightarrow \frac{11}{18}$$

$$⑳ ③ \quad \lim_{x\to 0} \frac{e^{\sinh(x)}-e^{x}}{(x+2)\left(\arctan(x)\right)^3} = \frac{0}{0}$$

$$=\frac{*}{(x+2)\cdot\left(x+o(x)\right)^3}=\frac{*}{2x^3+o(x^3)}$$

$$e^{\sinh(x)} = 1+\sinh(x)+\frac{1}{2}\left(\sinh(x)\right)^2+\frac{1}{6}\left(\sinh(x)\right)^3+o\left(\left(\sinh(x)\right)^3\right)$$

$$=1+\left(x+\frac{1}{6}x^3+o(x^3)\right)+\frac{1}{2}\left(x+\frac{1}{6}x^3+o(x^3)\right)^2+$$

$$+\frac{1}{6}\left(x+\frac{1}{6}x^3+o(x^3)\right)^3+o\left(\left(x+\frac{1}{6}x^3+o(x^3)\right)^3\right)$$

$$=1+x+\frac{1}{6}x^3+o(x^3)+\frac{1}{2}x^2+o(x^3)+\frac{1}{6}x^3+o(x^3)+o(x^3)$$

$$=1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+o(x^3)$$

$$\frac{e^{\sinh(x)} - e^{x}}{\dots} = \frac{\cancel{1+x+\frac{1}{2}x^2} + \frac{1}{3}x^3 + o(x^3) - \left(\cancel{1+x+\frac{1}{2}x^2} + \frac{1}{6}x^3 + o(x^3)\right)}{2x^3 + o(x^3)}$$

$$= \frac{1}{12}$$

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29 1 Trovare max/min rel. per

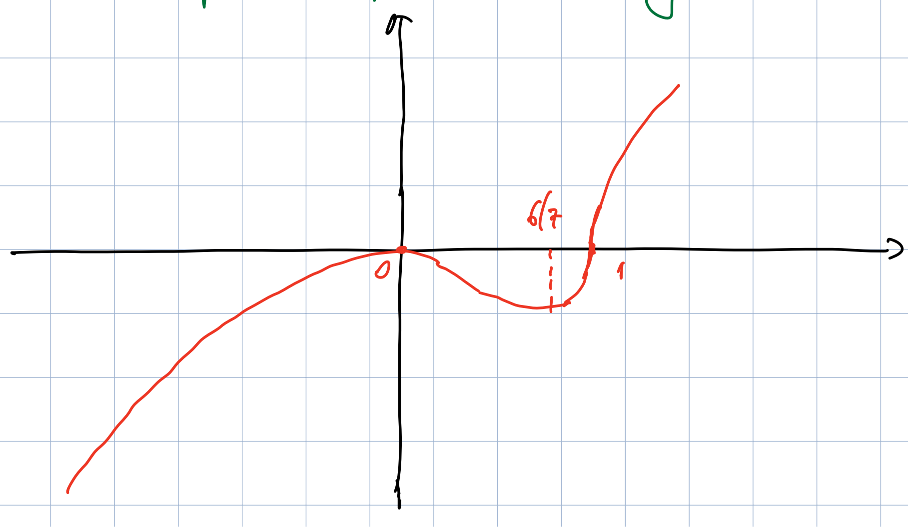
$$f(x) = x^2 \cdot \sqrt[3]{x-1}$$

$f : \mathbb{R} \to \mathbb{R}$ $\qquad \lim_{x \to \pm\infty} f(x) = \pm\infty$

$$f'(x) = 2x\sqrt[3]{x-1} + x^2 \cdot \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(x-1)^2}}$$

$$= \frac{x}{3 \cdot \sqrt[3]{(x-1)^2}} \cdot \left(6(x-1) + x\right) = \frac{x(7x-6)}{3 \cdot \sqrt[3]{(x-1)^2}}$$

$x = 1$ è pto di flesso a tangente verticale.



Ese sulle serie Zanichelli p. 244

③ $\sum \frac{n!}{n^n}$

Richiamo: se $\frac{a_{n+1}}{a_n} \to L$ oppure $\sqrt[n]{a_n} \to L$

$L < 1 \Rightarrow$ converge $\quad (a_n > 0)$

$L > 1 \Rightarrow$ diverge

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \frac{\frac{n!}{(n+1)^n}}{\frac{n!}{n^n}} = \frac{n^n}{(n+1)^n}$$

$$= \left(\frac{n}{n+1}\right)^n = \left(\left(\frac{n+1}{n}\right)^n\right)^{-1} = \left(\left(1+\frac{1}{n}\right)^n\right)^{-1}$$

$$\to e^{-1} = \frac{1}{e} < 1 \Rightarrow \text{convergente.}$$

④ $\sum \frac{\sqrt[n]{n}}{n!}$

$$\frac{\sqrt[n]{n}}{n!} \leq \frac{n}{n!} = \frac{1}{(n-1)!} \Rightarrow \text{converge per confronto.}$$

⑤ $\sum \frac{2^n}{e^{2n}} = \sum \left(\frac{2}{e^2}\right)^n$ (geometrica)

$$\frac{2}{e^2} < 1 \Rightarrow \text{converge}$$

⑥ $\sum \log\left(\frac{n+2}{n+4}\right)$ (Serie a termini negativi)

$$a_n = \log\left(\frac{n+4-2}{n+4}\right) = \log\left(1 - \frac{2}{n+4}\right) \simeq -\frac{2}{n+4}$$

il rapporto ha limite 1

Ricordo: $\sum \frac{1}{n^\alpha}$ diverge per $\alpha \leq 1$
converge per $\alpha > 1$ (visto per $\alpha \geq 2$)

$\Longrightarrow$ diverge a $-\infty$

⑦ $\sum \cos\left(\frac{n+2}{n^2+4}\right)$

$$a_n = \cos\left(\frac{n+2}{n^2+4}\right) \longrightarrow 1$$

$\Longrightarrow$ diverge a $+\infty$.

⑧ $\sum \log\left(\frac{n^2+2}{n^2-2}\right)$

$$\log\left(\frac{n^2+2}{n^2-2}\right) = \log\left(1 + \frac{4}{n^2-2}\right) \simeq \frac{4}{n^2-2}$$

$\Longrightarrow$ converge $\left(\sum \frac{1}{n^2} < +\infty\right)$

(9) $\sum \sin\left(\frac{n+2}{n^3+4}\right)$

$\sin\left(\frac{n+2}{n^3+4}\right) \sim \frac{n+2}{n^3+4} \simeq \frac{1}{n^2}$

$\Rightarrow$ Converge $\quad \left(\sum \frac{1}{n^2} < +\infty\right)$

Teoria: • $0 \leq a_n \leq b_n \quad \sum b_n < +\infty \Rightarrow \sum a_n < +\infty$

• $\frac{a_n}{b_n} \to L \neq 0$ allora $\sum a_n$ e $\sum b_n$ hanno stesso comportamento.

infatti: $a_n \leq \frac{3}{2} L \cdot b_n$ per n grande

$b_n \leq \frac{1}{2} L \cdot a_n$ per n grande

(10) $\sum \frac{(-1)^n}{\sqrt{n}}$ $\quad$ Sì

(Leibnitz: $a_n > 0$, $a_n \searrow 0$, $a_n \to 0$ $\Rightarrow \sum (-1)^n \cdot a_n$ converge)

Assolutamente convergente? $\quad \sum \frac{1}{\sqrt{n}}$ $\quad$ No

$\left(\sum \frac{1}{n^\alpha} \quad \alpha = 1/2\right)$

(11) $\sum_{n=1}^{\infty} \frac{\cos(n \cdot \frac{\pi}{2})}{n} = \sum_{m=1}^{\infty} \frac{(-1)^m}{2m} = \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)^m}{m}$

Converge (Leibnitz)

(12) $$\sum \frac{\sin(\log(n))}{n^2 \cdot \log(n)}$$

$$\left| \frac{\sin(\log(n))}{n^2 \cdot \log(n)} \right| \leq \frac{1}{n^2}$$

$\Rightarrow$ ass. conv $\left(\text{confr. con } \sum \frac{1}{n^2} < +\infty \right)$

$\Rightarrow$ conv.

(13) $$\sum \frac{\sin(n) + (-1)^n \cdot n}{n^2}$$

vuo che è a segno alterno
(per n grande, n prevale su sin(n))

prob. vuo che |...| ↘
difficile da formalizzare.

$$\sum \left(\frac{\sin(n)}{n^2} + \frac{(-1)^n}{n} \right)$$

ass. conv.

$$\left| \frac{\sin(n)}{n^2} \right| \leq \frac{1}{n^2}$$

converge per Leibnitz

$\Rightarrow$ converge