FOGLIO 2

4d $\left(2^{x}\right)^{y-\sqrt{3}} \cdot 5^{xy} \cdot \left(4^{x}\right)^{\sqrt{3}} : \left(10^{y}\right)^{x} =$

$2^{x(y-\sqrt{3})} \cdot 5^{xy} \cdot 2^{2\sqrt{3}x} : \left(2^{xy} \cdot 5^{xy}\right) =$

$2^{xy-\sqrt{3}x} \cdot 5^{xy} \cdot 2^{2\sqrt{3}x} \cdot 2^{-xy} \cdot 5^{-xy} =$

$2^{\cancel{xy}-\sqrt{3}x+2\sqrt{3}x-\cancel{xy}} \cdot 5^{\cancel{xy}-\cancel{xy}} = 2^{\sqrt{3}x}$ □

5a $\left(\left(3^{\log_3 x}\right)^2 - 2 \cdot 5^{\log_5 (x-1)} + \log_{\frac{1}{6}} 6\right) : \left(4^{\log_4 (x-1)}\right) =$

$\underbrace{\log_a \left(a^x\right) = x}$ $\underbrace{a^{\log_a x} = x}$

$\left(x^2 - 2 \cdot (x-1) + \log_{\frac{1}{6}}\left(\left(\frac{1}{6}\right)^{-1}\right)\right) : (x-1) =$

$\left(x^2 - 2x + 2 - 1\right) : (x-1) =$

$\left(\underbrace{x^2 - 2x + 1}\right) : (x-1) = (x-1)^2 : (x-1) = x-1$ □

5b $\log_{(a^{1/x^2})}\left(a^{\frac{1}{x-y}}\right) + \log_{(a^{y-x})}\left(a^{y^2}\right) =$

TRUCCO: $a^{\frac{1}{x-y}} = a^{\frac{1}{x^2} \cdot \frac{x^2}{x-y}}$ ✓

$\log_{(a^{1/x^2})}\left(\left(a^{\frac{1}{x^2}}\right)^{\frac{x^2}{x-y}}\right) + \log_{(a^{y-x})}\left(\left(a^{y-x}\right)^{\frac{y^2}{y-x}}\right) \circled{=}$

$$\frac{x^2}{x-y} + \frac{y^2}{y-x} = \frac{x^2-y^2}{x-y} = \frac{(x-y)(x+y)}{x-y} = x+y \quad \square$$

$(a^x)^y = a^{xy}$ PROPRIETÀ

$$\left(a^{\frac{1}{x^2}}\right)^{\frac{x^2}{x-y}} = a^{\frac{1}{x^2}\cdot\frac{x^2}{x-y}} = a^{\frac{1}{x-y}}$$

$a^{\frac{1}{x-y}}$ È UNA POTENZA DI $a^{\frac{1}{x^2}}$ ✓

5c

$$\left(\log_3(2) - \log_2(3)\right)\cdot\frac{\log_2(27)}{\log_3(\sqrt{3}) + \log_4(3)} =$$

$$\left(\log_3(2) - \log_2(3)\right)\cdot\frac{\log_2(3^3)}{\log_3(3^{\frac{1}{2}}) + \log_{2^2}(3)} =$$

FORMULA

$$\log_{a^k}(x) = \frac{1}{k}\log_a(x)$$

PORTIAMO TUTTI I LOG IN BASE 2

$$\left(\frac{\log_2(2)}{\log_2(3)} - \log_2(3)\right)\cdot\frac{\log_2(3^3)}{\frac{1}{2} + \frac{1}{2}\log_2(3)} =$$

$$\log_a(x^k) = k\log_a(x)$$

$$\left(\frac{1}{\log_2(3)} - \log_2(3)\right)\cdot\frac{3\log_2(3)}{\frac{1}{2}\left(1+\log_2(3)\right)} =$$

$$\left(\frac{1-(\log_2(3))^2}{\log_2(3)}\right)\cdot 6\,\frac{\log_2(3)}{1+\log_2(3)} =$$

$$\left(1-\log_2(3)\right)\left(1+\log_2(3)\right)\cdot 6\cdot\frac{1}{1+\log_2(3)} = 6 - 6\log_2(3) \quad \square$$

6a NUMERI COMPLESSI $z = a + ib$ $a, b \in \mathbb{R}$ $i^2 = -1$

$a = \mathrm{Re}(z)$ $b = \mathrm{Im}(z)$

$z = (2 + 11i) + (5 - 3i) = 7 + 8i$

$\mathrm{Re}(z) = 7$ $\mathrm{Im}(z) = 8$

$\mathrm{Re}(5i) = 0$

$\mathrm{Im}(37) = 0$

6b $z = (8 - 3i)(5 + 4i) = 40 + 32i - 15i - 12i^2$

$= 40 + 17i + 12 = 52 + 17i$

$\mathrm{Re}(z) = 52$ $\mathrm{Im}(z) = 17$

6d $z = (7 - 3i)^{-1} = \frac{1}{7 - 3i} \cdot \frac{7 + 3i}{7 + 3i} = \frac{7 + 3i}{49 - 9i^2}$

$= \frac{7 + 3i}{58} = \frac{7}{58} + \frac{3}{58} i$

$\mathrm{Re}(z) = \frac{7}{58}$ $\mathrm{Im}(z) = \frac{3}{58}$

$z^{-1} = \frac{\bar{z}}{|z|^2}$