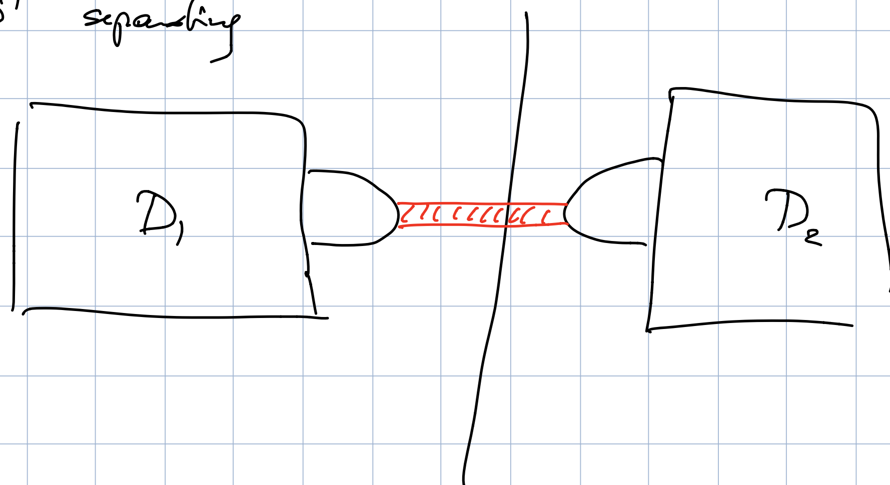
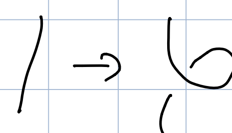
Teoria dei Nodi

4/4/19

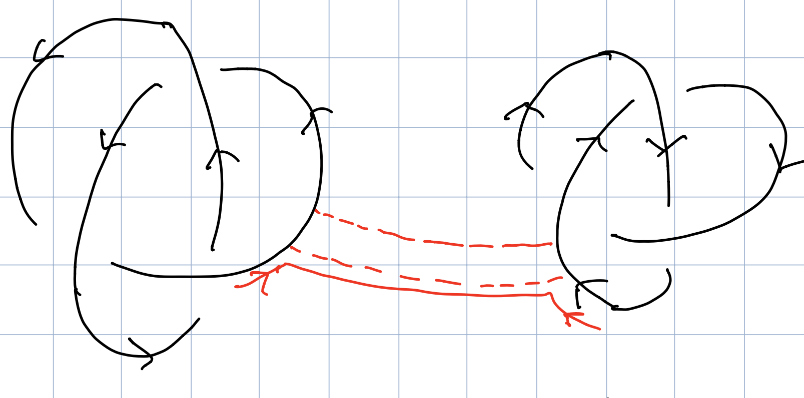
Version of # for diagrams: $D_1, D_2 \subset \mathbb{R}^2 \subset S^2$

$\gamma \cong S^1$ separating



maybe necessary to either add 

or use  :



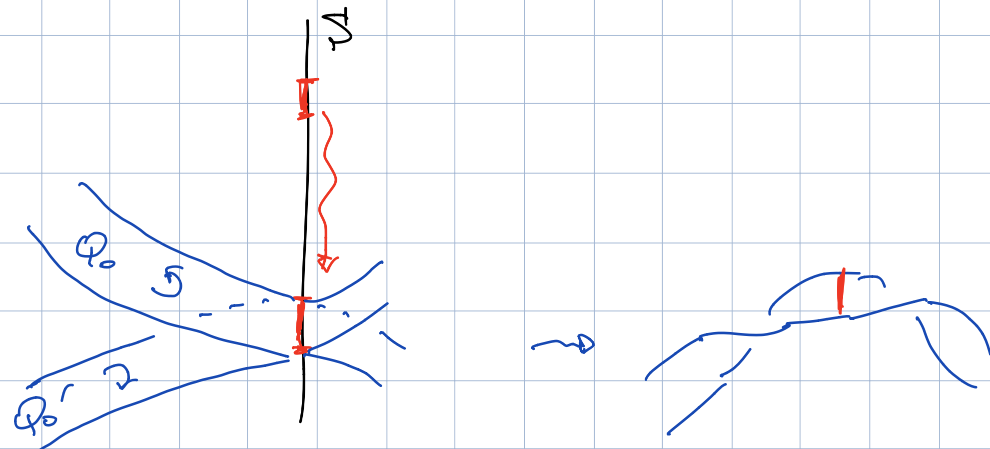
Prop: $K_0 \# K_1$ well-defined for oriented.

Proof: choices: • sphere S separating K_0 from K_1 ✓

• position of $K_j \subset B_j$ / isotopy ✓

• square :

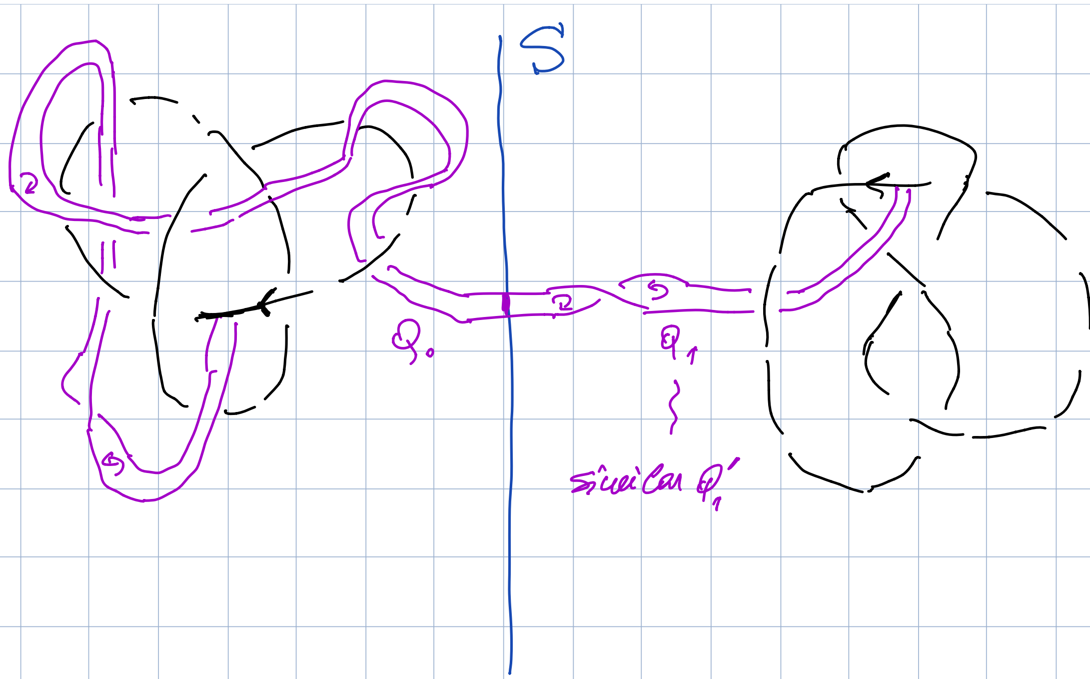
Suppose I use Q, Q' squares for $\#$.
Wlog can assume $Q \cap S = Q' \cap S = e$
(all segments on S are isotopic) and
if $Q_0 = Q \cap B_0$ $\quad Q_0' = Q' \cap B_0$ then
e is induced the same orientation by Q_0, Q_0'



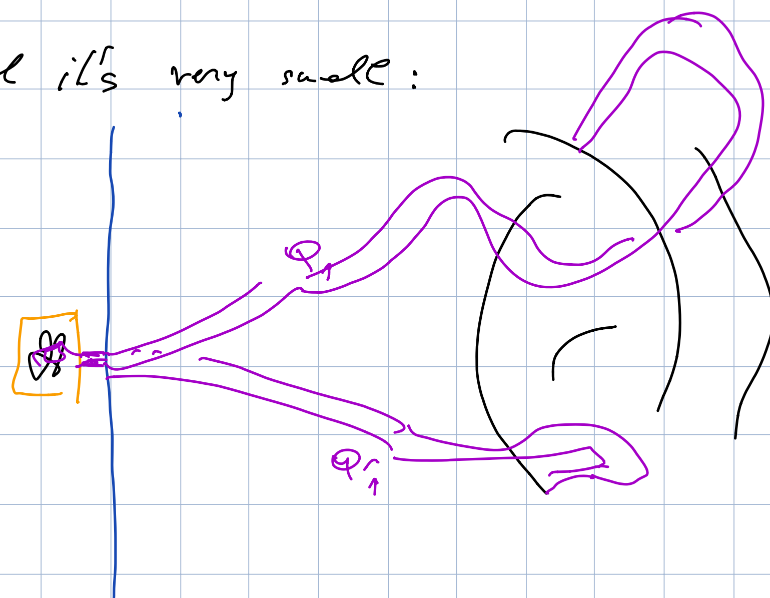
$Q_1 = Q \cap B_1$, $Q_1' = Q' \cap B_1$

Claim: using $Q_0 \cup Q_1 = Q$ or $Q_0 \cup Q_1'$ gives same
using $Q_0' \cup Q_1' = Q'$ or $Q_0 \cup Q_1'$ gives same

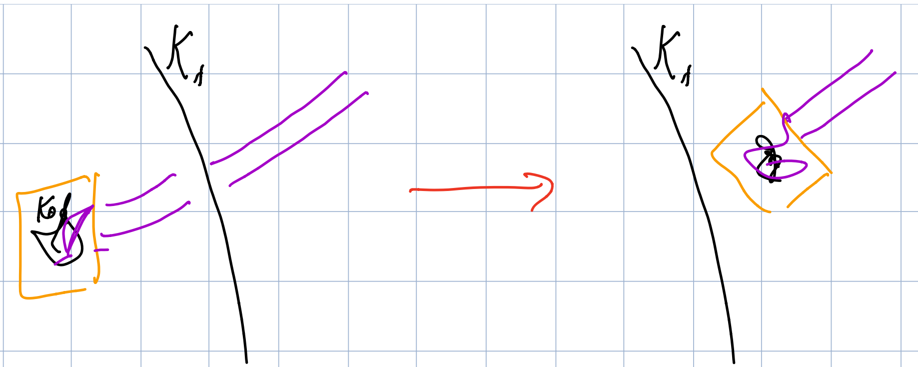
symmetric statements that imply conclusion.



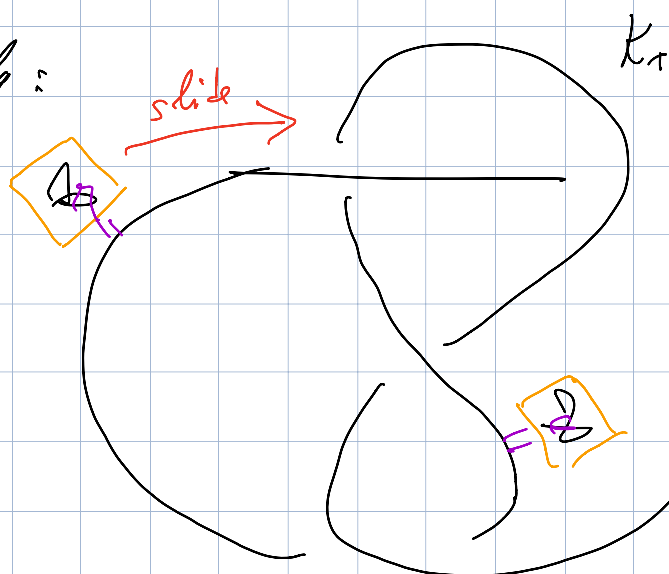
shrink LHS until it's very small:



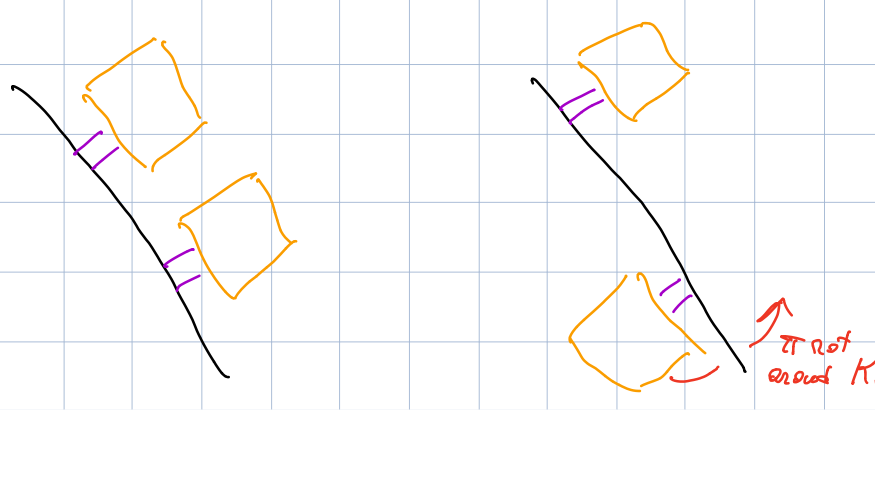
Slide flag along φ_1, φ_1' (I contract them to their end on K_1



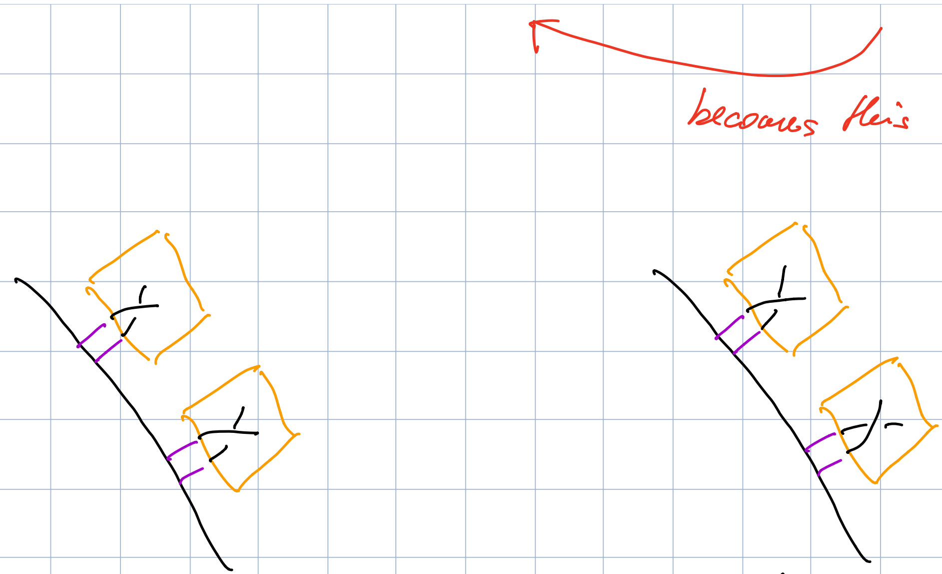
Eventually:



Eventually



becomes this



exercise: show this cannot happen because ∂'s induce good orientation on K. $\blacksquare$

<u>Will show</u>: $\exists$ well-defined decomposition of each knot as a $\#$ of <u>prime</u> knots (knots that do not decompose as $\#$ of two non-trivial knots).

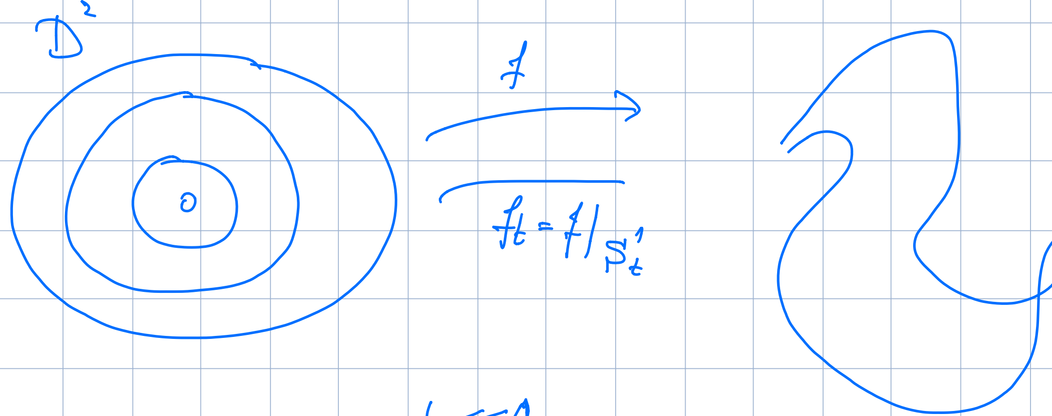
<u>Def</u>: $g(K)$ genus

$$= \min\{ g(\hat{\Sigma}) : \Sigma \text{ Seifert for } K,\ \hat{\Sigma} = \Sigma \cup D^2 \}$$

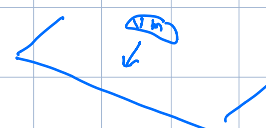
Unknot: unit circle in $\mathbb{R}^2 \times \{0\}$.

Rem: $g(K)=0 \Rightarrow K$ unknot.

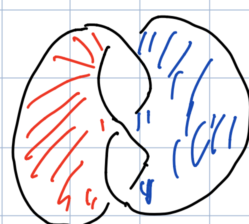
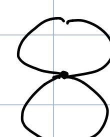
$g(K)=0 \longrightarrow K=\partial D \qquad D\cong D^2$ smooth



$t \ll 1$
almost planar



trefoil:

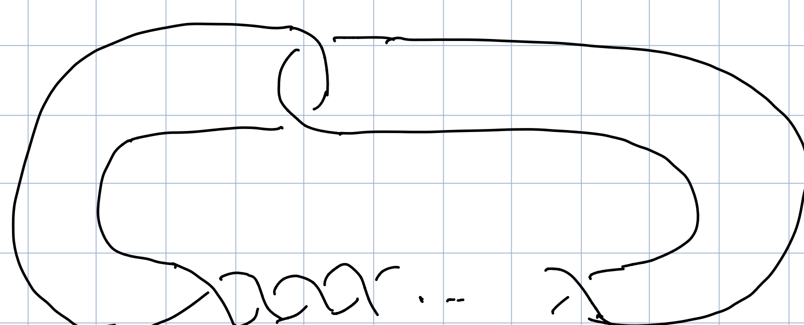
 $\simeq$ 

$\chi = 1-2 = -1 \qquad \chi = 2(1-g)-1$

$\Rightarrow g = 1$

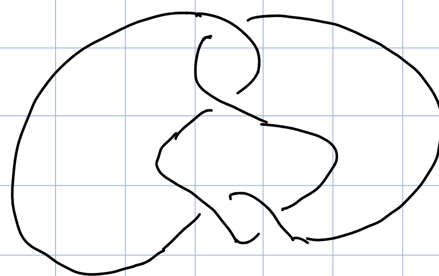
$\Rightarrow g(\text{tref}) = 1.$

<u>twist knots</u>:



$k \geq 1$ extra crossings

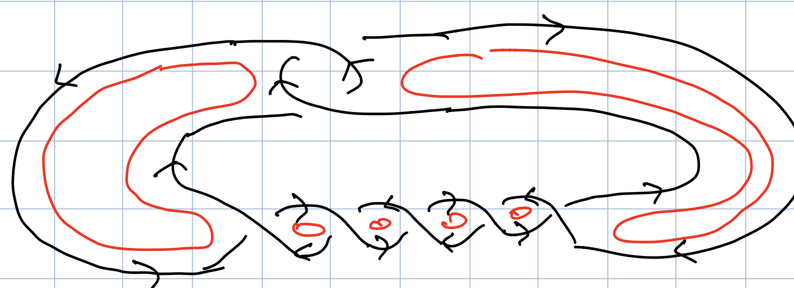
$k=1$ trefoil

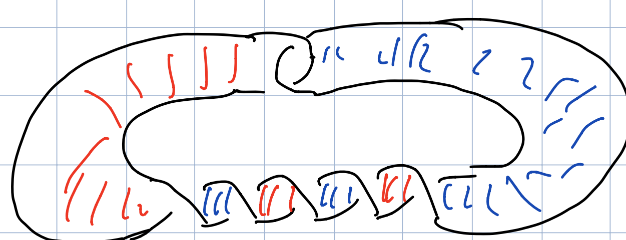


$k=2$ figure-8

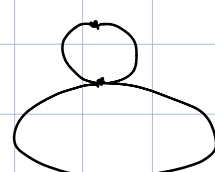
<u>Claim</u>: all have genus 1.

<u>odd k</u>:



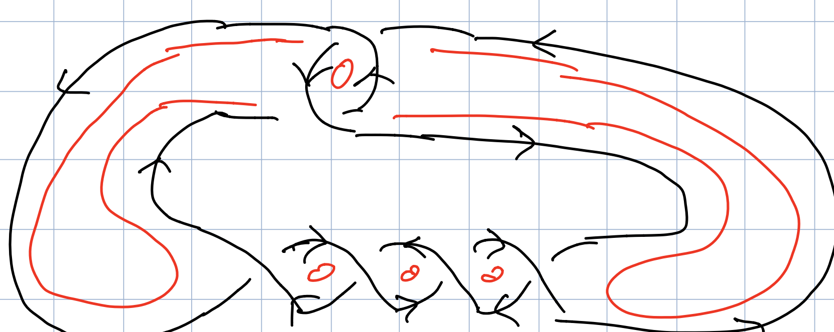


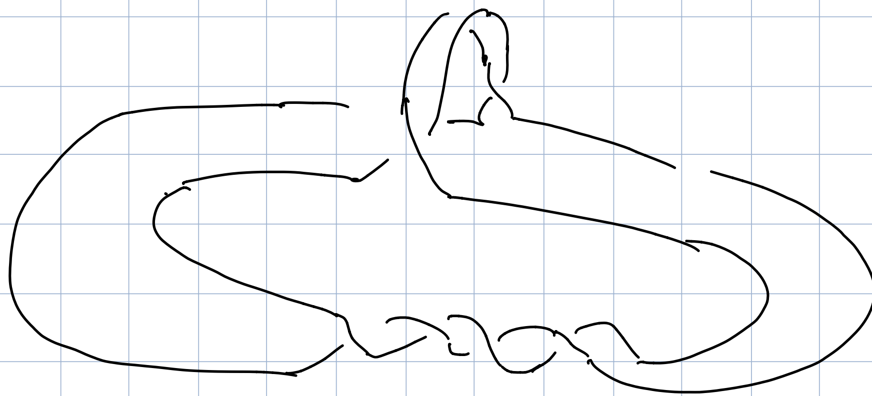
$\simeq$

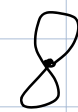


$\Longrightarrow g=1$

even k



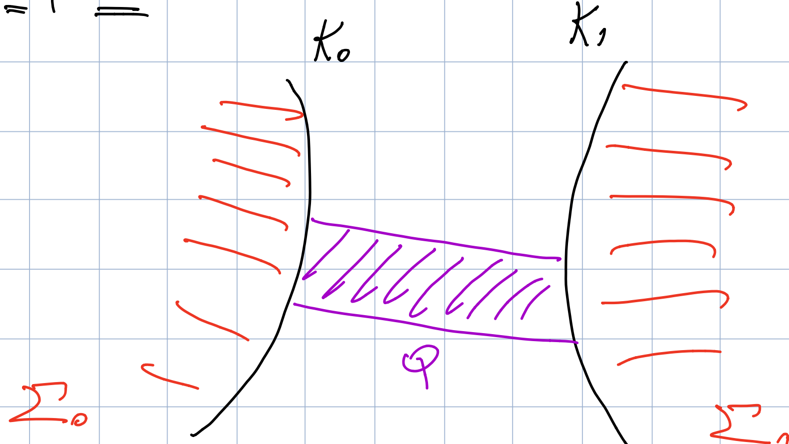


$\simeq$ 

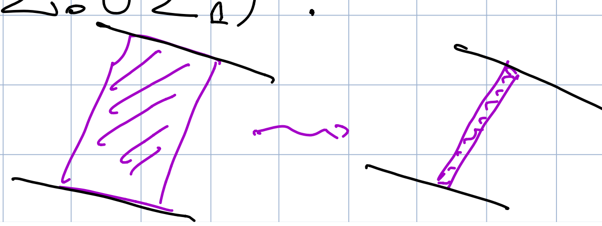
$\Rightarrow g = 1$

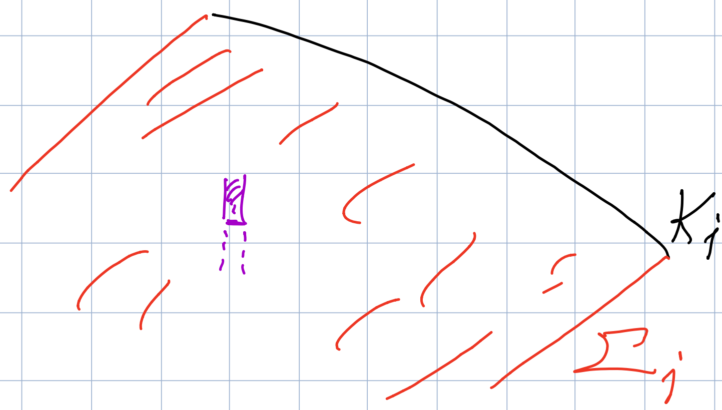
Thm: $g(K_0 \# K_1) = g(K_0) + g(K_1)$.

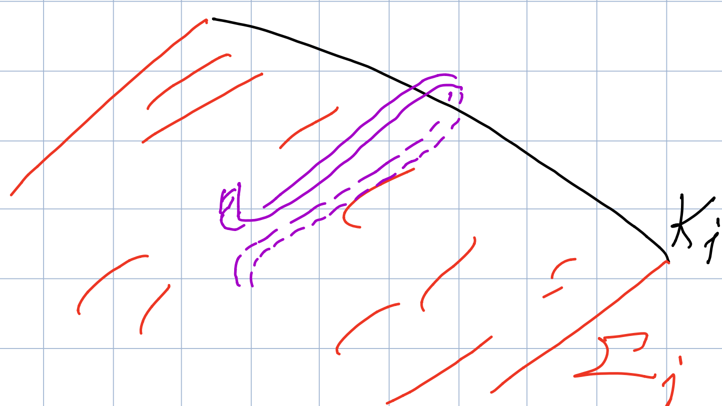
Proof of $\leq$



Fact: wlog $\varphi \cap (\Sigma_0 \cup \Sigma_1)$:







$\Longrightarrow Q \cup \Sigma_0 \cup \Sigma_1$ Seifert for $K_0 \# K_1$

$$\chi(\Sigma) = \chi(\Sigma_0) + \chi(\Sigma_1) - 1$$

$$2(1-g) - 1 = 2(1-g_0) - 1 + 2(1-g_1) - 1 - 1$$

$$\Longrightarrow g = g_0 + g_1.$$

$\boxtimes$