IDEAL LIMITS OF SEQUENCES OF CONTINUOUS FUNCTIONS AND A GAME

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The results presented here are included in paper Miklós Laczkovich, Ireneusz Recław *Ideal limits of sequences of continuous functions.*
We say that $J - \lim x_n = x$ if for each $\epsilon > 0$, $\{n : |x - x_n| > \epsilon\} \in J$. We say that the function $f : X \to \mathbb{R}$ is J-limit of sequence of functions $< f_n : n \in \omega >$ if for each $x \in X$, $f(x) = J - \lim f_n(x)$. The class of all J limits of $C(X)$ is denoted by $J - \lim C(X)$. It is known that J-limit of continuous functions can be nonmeasurable for maximal ideals (foklore) or of arbitrary high Baire class for Borel ideals (Katětov). Kostyrko, Šalát, Wilczyński proved that for the ideal of sets of density zero $J - \lim C(X) = B_1(X)$. We investigate ideals for which $J - \lim C(X) = B_1(X)$.

Let us define an infinite game $G(J)$ where J is an ideal on the integers. Player I in the n$^{\text{th}}$ move plays an elements C_n of the ideal, and then player II plays a finite subsets of integers F_n with $F_n \cap C_n = \emptyset$. Player I wins when $\bigcup_n F_n \in J$. Otherwise player II wins. This game was investigated by C. Laflamme.

Theorem 1.

(1) *Let X be a complete metric space. Assume that player II has a winning strategy in $G(J)$. Then $J - \lim C(X) = B_1(X)$.*
(2) *Let X be a topological space. Assume that player I has a winning strategy in $G(J)$. Then $B_2(X) \subset J - \lim C(X)$.*

By Borel determinacy we obtain:

Theorem 2. *Assume that J is a Borel ideal and X is a complete metric space. Then $J - \lim C(X) = B_1(X)$ iff J does not contain a copy of $FIN \times FIN$.*

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