ULTRAPRODUCTS AND LIE ALGEBRAS: SOME POSSIBLE INTERACTIONS

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This report aims at illustrating the role played by the ultraproducts in two joint works, the former with Angus Macintyre [1] and with Francoise Point and Angus Macintyre [2], both related to a model theoretic investigation of certain representations of the universal enveloping algebra U_k of $sl_2 k$ (the Lie algebra of trace zero 2×2 matrices over an algebraically closed field k of characteristic zero).

In [1], we investigate the theory of all pseudo-finite dimensional representations of U_k, (meaning infinite dimensional representations satisfying in the language of U_k-modules the same first order sentences as finite dimensional representations of U_k). By the fact that any pseudo-finite dimensional representation is elementarily equivalent to a ultraproduct of finite dimensional modules, we can bring out a connection between the decidability of this theory and some fundamental problems in the diophantine geometry of curves.

In [2], by restricting our attention to the field of complex numbers, we try to define an exponential map over the universal enveloping algebra of $sl_2\mathbb{C}$. We will emphasize the application of the ultraproducts to this matter.

REFERENCES

[1] S. L'Innocente, A. Macintyre, Towards decidability of the theory of pseudo-finite dimensional representations of $sl_2 k$; I, In: A. Ehrenfeucht, V.W. Marek, M. Srebrny. Andrzej Mostowski and Foundational Studies. *IOS Press*, 2008, 235-260

[2] S. L'Innocente, A. Macintyre, F. Point, Possible exponentiations over the universal enveloping algebra of $sl_2\mathbb{C}$, *In progress*.

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