A DEFINABLE FRAMEWORK FOR NONSTANDARD ANALYSIS

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The present paper establishes the existence of a definable (over ZFC), countably saturated nonstandard enlargement of the superstructure over the reals.

V. Kanovei and S. Shelah [*A definable nonstandard model of the reals.* Journal of Symbolic Logic **69**, No. 1, pp. 159–164 (2004)] constructed an ultrafilter that mimics the idea of a finite iteration of ultrafilters and used this object to obtain a definable (over ZFC) model of the reals. However, in order to have a definable framework for fully-fledged nonstandard analysis, one would need to find a definable (countably) saturated nonstandard enlargement of the superstructure over the reals.

Extending the methodology of Kanovei and Shelah [*loc. cit.*] to *bounded* ultrapowers (i.e. bounded with respect to the superstructure hierarchy), this paper shows that the desired nonstandard universe can be obtained as the union of an inductive chain of bounded ultrapowers, taken with respect to the original ultrafilter employed by Kanovei and Shelah.

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