REAL PROJECTIVE STRUCTURES ON MANIFOLDS AND THE HYPERREALS

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[Joint work with Kelly Delp.]

We use the hyperreals to understand the limits of certain kinds of metrics on manifolds. The possible hyperbolic metrics (constant curvature -1) on a surface are paramaterized by points in an open ball called *Teichmuller space* whose dimension depends on the surface.

Thurston compactified Teichmuller space using projective classes of measured foliations. We give a new interpretation of these objects as non-standard hyperbolic structures on the surface defined over the hyperreals.

We then generalize this by studying limits of convex real projective structures on a compact n-manifold (or orbifold) M. Such a structure gives rise to a Finsler metric on M called the Hilbert metric. For 2-manifolds, given a sequence of such structures, after suitable rescaling and subsequencing, one obtains a limiting structure on M which gives a measured foliation on a subsurface and a singular HeX structure (a particularly nice Finsler metric coming from a certain norm) on the complementary subsurface.

Another view of this is obtained by doing projective geometry over the hyperreals. The ultrapower of the fundamental group of the surface acts on a convex domain in the projective plane over the hyperreals (an ultralimit of convex domains in the standard projective plane) and the quotient is a non-standard projective surface. A sequence of convex projective structures on a surface M defines a nonstandard (over the hyperreals) projective structure on M. It has a Finsler metric taking values in the hyperreals. One decomposes this surface into a thin part where the injectivity radius is infinitesimal and the complementary thick part. An interpretation of this structure in the standard setting, over the reals, yields the above decomposition of M. This is closely related to the geometry of the asymptotic cone of PGL(n+1,R).

The core mathematics involves looking at a convex finite sided polytope in the n-dimensional vector space over the hyperreals equipped with a particular metric taking values in the hyperreals, and face pairings by nonstandard isometries to obtain a nonstandard manifold. Then one must interpret this data on a standard version of the manifold. One important issue is that the topology of the hyperreals is not good for studying manifolds, and to overcome this we use *piecewise linear topology* which does work well in the hyperreal context.

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