ON PSEUDO–INTERSECTIONS AND CONDENSERS

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[Joint work with Grzegorz Plebanek.]

A set $A \subseteq \mathbb{N}$ is a pseudo–intersection of a filter $\mathcal{F}$ on $\mathbb{N}$ if A is contained in every element of $\mathcal{F}$ modulo a finite set. In a similar way we can define a condenser of a filter $\mathcal{F}$, i.e. a set A such that every element of $\mathcal{F}$ is of density 1 in A. Every pseudo–intersection is a condenser and a filter of density 1 sets is an example of a filter without pseudo–intersection but with a condenser. One can ask, if it is possible to construct a Boolean algebra $\mathbb{A}$ such that no ultrafilter on $\mathbb{A}$ has a pseudo–intersection but every ultrafilter possesses a condenser. It is easy that under CH such algebra does not exist, but it is unclear if it can be shown in ZFC. We will show that the existence of such an algebra would be implied by a certain inequality on cardinal invariants. Unfortunately it is not known if this inequality is consistent with ZFC. Also, we show that under an additional set–theoretic assumption there is a Boolean algebra $\mathbb{A}$ such that no ultrafilter on $\mathbb{A}$ has a pseudo–intersection, but each ultafilter is feeble (i.e. it can be embedded in the Frechet filter, in a certain way). We indicate some connections of the subject with a problem from functional analysis.

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