STOCHASTIC NAVIER-STOKES EQUATIONS: IDEAS AND RESULTS USING NONSTANDARD ANALYSIS

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The Navier-Stokes equations in 3-dimensions, which give a mathematical model of fluid flow in a region of space, are still not fully understood: settling the problem of existence of a strong solution for all time would net one of the Millennium prizes. The corresponding problem for weak solutions was settled in the 1930's by Leray.

This talk will survey results concerning weak solutions for the stochastic Navier-Stokes equations in 2 or 3 dimensions that have been achieved, in collaboration with Marek Capinski, Jerry Keisler, Kasia Grzesiak and Brendan Enright, using ideas from nonstandard analysis, and in particular Loeb measures. These include basic existence results as well as results on attractors and control theory for these equations, and an extension to the non-homogeneous (i.e. non-constant density) equations. The emphasis will be the underlying ideas behind the proofs rather than technical details. We will aim to explain how nonstandard analysis lends itself naturally to the study of the Navier-Stokes equations when they are formulated as a differential equation in a suitable separable Hilbert space. At the nonstandard level the equations become a hyperfinite dimensional system and so all the results of finite dimensional DE and SDE theory become available through the Transfer Principle.

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