

ULTRAFILTERS AND ERGODIC RAMSEY THEORY

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The talk will concentrate on various applications of ultrafilters in Ramsey theory, topological dynamics and Ergodic theory. For example, the classical by now Hindman's theorem which states that for any finite partition of \mathbb{N} , one cell of the partition contains an infinite set together with all finite sums formed by its distinct elements, can be given a very short proof based on the existence of idempotent ultrafilters in the Stone-Chech compactification of \mathbb{N} . More modern applications of ultrafilters are related with some refinements and extensions of the celebrated Szemerédi theorem which states that any set of positive upper density in \mathbb{N} contains arbitrarily long arithmetic progressions. We will discuss a new extension of the Szemerédi's theorem (joint work with Randall McCutcheon) which utilizes minimal idempotent ultrafilters and "upgrades" the polynomial extension of Szemerédi's theorem, obtained some time ago jointly with Alexander Leibman, to families of generalized polynomials, namely functions which are obtained from regular polynomials via iterated use of the floor function. The last part of the talk will be devoted to open problems and conjectures.

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