Ulteriori proprietà del valore assoluto

- $|a| \geq 0, \ \forall a \in \mathbb{R}$; $|a| = 0 \iff a = 0$

- $|a| = |b| \iff a = b$ oppure $a = -b$

- $|a| \leq b \iff -b \leq a \leq b \quad (\forall a \in \mathbb{R}, \ \forall b > 0)$
- $|a| \geq b \iff a \leq -b \vee a \geq b \quad (\forall a \in \mathbb{R}, \ \forall b > 0)$

- $|a| \leq |b| \iff a^2 \leq b^2 \quad (\forall a, b \in \mathbb{R})$

Equazioni e disequazioni coni i valori assoluti

- $|3x + 5| = 0 \iff 3x + 5 = 0$; $x = -\frac{5}{3}$

- $|3x + 7| = 2 \iff \left[\begin{array}{l} 3x + 7 = 2 \\ \text{oppure} \\ 3x + 7 = -2 \end{array}\right.$ $\quad x = -\frac{5}{3} \vee x = -3$

- $|2x + 1| = -3$, $\emptyset$

- $|x^2 - 4x| = 4 \iff \left[\begin{array}{l} x^2 - 4x = 4 \\ \text{oppure} \\ x^2 - 4x = -4 \end{array}\right.$ $\quad x = 2 \pm 2\sqrt{2} \ \vee \ x = 2$

- $|3x - 2| < 1 \iff \begin{cases} 3x - 2 < 1 \\ 3x - 2 > -1 \end{cases}$ $\quad \frac{1}{3} < x < 1$

- $|x^2 - 5x| \geq 6 \iff x^2 - 5x \geq 6$ oppure $x^2 - 5x \leq -6$ $\quad x \leq -1 \vee 2 \leq x \leq 3 \vee x \geq 6$

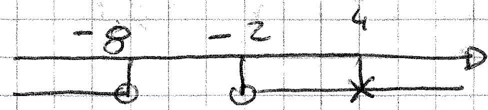
- $|3x - 1| \leq |x + 2| \iff (3x - 1)^2 \leq (x + 2)^2$; $-\frac{1}{4} \leq x \leq \frac{3}{2}$

• $\left|\dfrac{x}{2x+3}\right| > 1 \iff \dfrac{x}{2x+3} > 1 \qquad -3 < x < -\dfrac{3}{2}$

oppure $\qquad \vee$

$\dfrac{x}{2x+3} < -1 \qquad -\dfrac{3}{2} < x < -1$

• $\dfrac{3 - |x+5|}{|x-4|} \leq 0$



$x \leq -8 \ \vee\ -2 \leq x < 4 \ \vee\ x > 4$

6)

Disequazioni irrazionali

$$\sqrt{f(x)} < g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) > 0 \\ f(x) < [g(x)]^2 \end{cases}$$

$$\sqrt{f(x)} \leq g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ f(x) \leq [g(x)]^2 \end{cases}$$

$$\sqrt{f(x)} > g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ f(x) > [g(x)]^2 \end{cases}$$

$$\sqrt{f(x)} \geq g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ f(x) \geq [g(x)]^2 \end{cases}$$

$\sqrt{3x+1} < 2x-1$; $x \in \left]\frac{7}{4}; +\infty\right[$; $\boxed{x > \frac{7}{4}}$

$\sqrt{3-2x} \leq x+2$; $x \in \left[-3+2\sqrt{2}; \frac{3}{2}\right]$; $\boxed{-3+2\sqrt{2} \leq x \leq \frac{3}{2}}$

$\sqrt{3x-2} > 2(x-1)$; $x \in \left[\frac{2}{3}; 2\right[$; $\boxed{\frac{2}{3} \leq x < 2}$

$\sqrt{x^2-4} - 2x + 5 \geq 4 - x$; $x \in \left]-\infty; -2\right] \cup \left[\frac{5}{2}; +\infty\right[$

$\boxed{x \leq -2 \text{ e } x \geq \frac{5}{2}}$

$\dfrac{6x-1-\sqrt{5-2x}}{x^2-4x+3} \leq 0$; $x \in \left]-\infty; \frac{1}{2}\right] \cup \left]1; \frac{5}{2}\right]$

$$\frac{x^2+5x}{2x-4+\sqrt{x+3}} \geq 0 \qquad [-3;0] \cup]1;+\infty[$$

$$-3 \leq x \leq 0 \quad \vee \quad x > 1$$

$\sqrt{2x+1} > 2$; $x \in [\frac{3}{2};+\infty[$; $x > \frac{3}{2}$

$\sqrt{x^2-4} < 1$; $x \in]-\sqrt{5};-2] \cup [2;\sqrt{5}[$

$$-\sqrt{5} < x \leq -2 \quad \vee \quad 2 \leq x < \sqrt{5}$$

$\sqrt{3x+1} < -3$; $\emptyset$

$\sqrt{2x+1} > -1$; $x \geq -\frac{1}{2}$; $x \in [-\frac{1}{2};+\infty[$

$\sqrt[3]{f(x)} < g(x) \iff f(x) < [g(x)]^3$

$\sqrt[3]{f(x)} > g(x) \iff f(x) > [g(x)]^3$

$\sqrt[3]{x^3+2x^2-1} \geq x+1$ $\quad x \in [-2;-1]$

$$-2 \leq x \leq -1$$

$\sqrt[3]{x^3-1} < (x+3)$; $\forall\, x \in \mathbb{R}$

$\sqrt[3]{x+1} < 2$; $]-\infty;7]$; $\boxed{x \leq 7}$

$x - \sqrt[3]{x^3-2x+1} \leq 0$; $]-\infty;\frac{1}{2}]$; $\boxed{x \leq \frac{1}{2}}$

Calcolare il valore dei seguenti logaritmi

$\log_3 27 = 3$; $\log_3 \frac{1}{81} = -4$; $\log_4 32 = \frac{5}{2}$

$\log_{10} 1 = 0$; $\log_{10} 0{,}01 = -2$; $\log_5 \frac{\sqrt[5]{5}}{5} = -\frac{4}{5}$

$\log_2 \sqrt[3]{16} = \frac{4}{3}$; $\log_{\sqrt{2}} \sqrt[5]{8} = \frac{6}{5}$; $\log_3 \frac{\sqrt[3]{9}}{3} = -\frac{1}{3}$

Determinare l'argomento dei seguenti logaritmi

$\log_{10} x = -1$; $x = \frac{1}{10}$; $\log_e x = \frac{2}{3}$ $\quad x = \sqrt[3]{e^2}$

$\log_3 x = 0$ $\quad x = 1$; $\log_2 x = -\frac{3}{2}$ $\quad x = \frac{\sqrt{2}}{4}$

$\log_{\sqrt{3}} x = 3$ $\quad x = 3\sqrt{3}$; $\log_{\sqrt{5}} x = -\frac{2}{3}$ $\quad x = \frac{\sqrt[3]{25}}{5}$

Determinare la base dei seguenti logaritmi

$\log_x 5 = +1$ $\quad x = 5$; $\log_x \frac{1}{2} = -1$ $\quad x = 2$

$\log_x \frac{1}{4} = -2$ $\quad x = 2$; $\log_x \frac{1}{2\sqrt{2}} = -3$ $\quad x = \sqrt{2}$

Trasformare in base e i seguenti logaritmi

$\log_2 \frac{5}{7} = \frac{\ln 5 - \ln 7}{\ln 2}$; $\log_{10} 9 = \frac{2\ln 3}{\ln 10}$

$\ln_{\frac{5}{3}} 3 = \frac{\ln 3}{\ln 5 - \ln 3}$; $\log_{\sqrt{5}} 2\sqrt{2} = \frac{3\ln 2}{\ln 5}$

Verificare le seguenti uguaglianze dopo aver stabilito per quali valori di x sono valide

$\ln(x^2+6x+9) = 2\ln(x+3)$ $\qquad x > -3$

$\ln(x^2+6x+9) = 2\ln|x+3|$ $\qquad x \neq -3$

$\ln\left(\frac{\sqrt{3-x}}{x^2-2}\right) = \frac{1}{2}\ln(3-x) - \ln(x^2-2)$; $\boxed{x < -\sqrt{2} \lor \sqrt{2} < x < 3}$

$\ln\sqrt{\frac{x^2+2x+1}{x^2-2x-1}} = \ln|x+1| - \frac{1}{2}\ln(x^2-2x-1)$

$x < -1 \lor -1 < x < -(\sqrt{2}-1) \lor x > 1+\sqrt{2}$