$$\int \frac{dx}{x^2+x+1} = \int \frac{4 \quad dx}{4x^2+4x+4} = \int \frac{4\,dx}{4x^2+4x+1+3} = \int \frac{4\,dx}{(2x+1)^2+3}$$

$\Delta = 1-4 = -3 < 0$

$$= \int \frac{4}{3} \frac{dx}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2} = \frac{\cancel{4}^{\,2}}{3} \frac{1}{\frac{\cancel{2}}{\sqrt{3}}} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{2}{3}\sqrt{3} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

Integrali definiti

$$\int_0^{\pi/4} tg^2 x \, dx = \int_0^{\pi/4} \left[(1+tg^2 x) - 1\right] dx = \left[tg x - x\right]_0^{\pi/4} = 1 - \frac{\pi}{4}$$

$$\int_0^3 |x^2-4| \, dx = \int_0^2 (4-x^2)\,dx + \int_2^3 (x^2-4)\,dx = \left[4x - \frac{x^3}{3}\right]_0^2 + \left[\frac{x^3}{3} - 4x\right]_2^3$$

$$|x^2-4| = \begin{cases} x^2-4, & x \leq -2 \vee x \geq 2 \\ 4-x^2, & -2 \leq x \leq 2 \end{cases}$$

$$= 8 - \frac{8}{3} + \cancel{\frac{27}{3}}^{\,9} - 12 - \frac{8}{3} + 8$$

$$= 13 - \frac{16}{3} = \frac{23}{3}$$

$$\int_0^1 \frac{x}{\sqrt{1-x^2}} \, dx = \lim_{k \to 1^-} \int_0^k \frac{x}{\sqrt{1-x^2}} \, dx = \lim_{k \to 1^-} \left[-\sqrt{1-x^2}\right]_0^k =$$

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = -\frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2} + c$$

$$= \lim_{k \to 1^-} \left(1 - \sqrt{1-k^2}\right) = 1$$

$$\int_0^1 x \log x \, dx = \lim_{\varepsilon \to 0^+} \int_\varepsilon^1 x \log x \, dx = \lim_{\varepsilon \to 0^+} \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_\varepsilon^1$$

$$\int x \log x \, dx = \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx = \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

$$= \lim_{\varepsilon \to 0^+} \left(-\frac{1}{4} + \frac{\varepsilon^2}{4} - \frac{1}{2} \varepsilon^2 \log \varepsilon\right) = -\frac{1}{4}$$

$$\lim_{\varepsilon \to 0^+} \varepsilon^2 \log \varepsilon = \lim_{\varepsilon \to 0^+} \frac{\log \varepsilon}{\frac{1}{\varepsilon^2}} \overset{H}{=} \lim_{\varepsilon \to 0} \frac{\frac{1}{\varepsilon}}{-\frac{2}{\varepsilon^{3}}} = \lim_{\varepsilon \to 0^+} -\frac{\varepsilon^2}{2} = 0$$

$$\int_1^{+\infty}(x^2-1)e^{-x}\,dx = \lim_{R\to+\infty}\int_1^{R}(x^2-1)e^{-x}\,dx$$

$$\int(x^2-1)e^{-x}\,dx = (ax^2+bx+c)e^{-x} = -(x+1)^2e^{-x}$$

$$D(ax^2+bx+c)e^{-x} = (x^2-1)e^{-x}$$

$$(-ax^2-bx-c+$$
$$+2ax+b)e^{-x} = (x^2-1)e^{-x}$$

$$[-ax^2+(2a-b)x+(b-c)]\cancel{e^{-x}} = (x^2-1)\cancel{e^{-x}}$$

$$\begin{cases}-a=1\\ 2a-b=0\\ b-c=-1\end{cases} \qquad \begin{cases}a=-1\\ b=2a=-2\\ c=b+1=-1\end{cases}$$

$$\lim_{R\to+\infty}\left[-(x+1)^2e^{-x}\right]_1^R = \lim_{R\to+\infty}\left[-(R+1)^2e^{-R}+\frac{4}{e}\right] = \frac{4}{e}$$

$$\lim_{R\to+\infty} -(R+1)^2e^{-R} = \lim_{R\to+\infty} -\frac{(R+1)^2}{e^R} \overset{H}{=} -\lim_{R\to+\infty}\frac{2(R+1)}{e^R} \overset{H}{=}$$

$$= -2\lim_{R\to+\infty}\frac{1}{e^R} = 0$$

Calcolare l'area dei seguenti domini piani

$$\begin{cases}y \le -x^2-3x+4\\ y \ge x^2+5x+4\end{cases}$$

$x^2+5x+4=0 \quad x=-4 \vee x=-1$

$x^2+3x-4=0 \quad x=-4 \vee x=1$

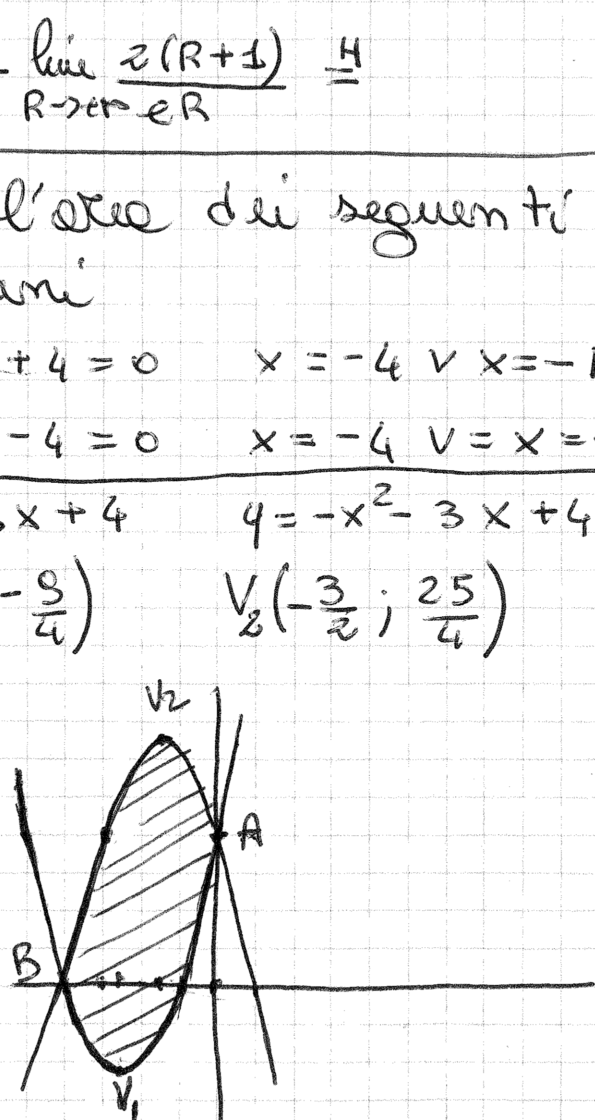
$y=x^2+5x+4 \qquad y=-x^2-3x+4$

$V_1\left(-\frac{5}{2};-\frac{9}{4}\right) \qquad V_2\left(-\frac{3}{2};\frac{25}{4}\right)$

$$\begin{cases}y=-x^2-3x+4\\ y=x^2+5x+4\end{cases}$$

$$\begin{cases}x^2+5x+\cancel{4}=-x^2-3x+\cancel{4}\\ y=x^2+5x+4\end{cases}$$

$$\begin{cases}2x^2+8x=0\\ y=x^2+5x+4\end{cases} \quad A\begin{cases}x=0\\ y=4\end{cases} \quad B\begin{cases}x=-4\\ y=0\end{cases}$$



$$\int_{-4}^{0}\left[(-x^2-3x+4)-(x^2+5x+4)\right]dx$$

$$= -\int_{-4}^{0}(2x^2+8x)\,dx = -\left[\frac{2}{3}x^3+4x^2\right]_{-4}^{0} =$$

$$-\frac{2}{3}64+4\cdot 16 = \frac{64}{3}\cdot$$

$$\int_0^{+\infty}\frac{dx}{1+x^2} = \lim_{R\to+\infty}\int_0^R \frac{dx}{1+x^2} = \lim_{R\to+\infty}\Big[\operatorname{arctg} x\Big]_0^R =$$

$$= \lim_{R\to+\infty} \operatorname{arctg} R = \frac{\pi}{2}$$

$$\int_0^{+\infty}\frac{x\,dx}{1+x^2} = \lim_{R\to+\infty}\frac{1}{2}\int_0^R \frac{2x}{1+x^2}dx = \lim_{R\to+\infty}\frac{1}{2}\Big[\log(1+x^2)\Big]_0^R$$

$$= \lim_{R\to+\infty}\frac{1}{2}\log(1+R^2) = +\infty$$

$$\int_0^R \operatorname{sen} x\,dx = \Big[-\cos x\Big]_0^R = 1-\cos R$$

Poichè $\nexists \lim (1-\cos R)$ per $R\to+\infty$

$\nexists \int_0^{+\infty}\operatorname{sen} x\,dx$ cioè $f(x)=\operatorname{sen} x$ non è integrabile nell'intervallo $[0;+\infty)$

$\boxed{\alpha>0}$ $$\int_0^1\frac{dx}{x^\alpha} = \lim_{\varepsilon\to 0^+}\int_\varepsilon^1 \frac{dx}{x^\alpha}$$

$$\int_\varepsilon^1\frac{dx}{x^\alpha} = \left[\frac{x^{-\alpha+1}}{-\alpha+1}\right]_\varepsilon^1 = \begin{cases} \dfrac{1}{1-\alpha}-\dfrac{\varepsilon^{1-\alpha}}{1-\alpha}, & 0<\alpha<1 \\[2ex] -\dfrac{1}{\alpha-1}+\dfrac{1}{\alpha-1}\dfrac{1}{\varepsilon^{\alpha-1}}, & \alpha>1\end{cases}$$

$\alpha\neq -1$

$$\int_\varepsilon^1\frac{dx}{x} = \Big[\log|x|\Big]_\varepsilon^1 = -\log\varepsilon$$

$$\int_0^1\frac{dx}{x^\alpha} = \begin{cases} \lim\limits_{\varepsilon\to0^+}\left(\dfrac{1}{1-\alpha}-\dfrac{\varepsilon^{1-\alpha}}{1-\alpha}\right) = \dfrac{1}{1-\alpha} & \text{per } 0<\alpha<1 \\[2ex] \lim\limits_{\varepsilon\to0^+} -\log\varepsilon = +\infty & \text{per } \alpha=1 \\[2ex] \lim\limits_{\varepsilon\to0^+}\left(-\dfrac{1}{\alpha-1}+\dfrac{1}{\alpha-1}\dfrac{1}{\varepsilon^{\alpha-1}}\right) = +\infty & \text{per } \alpha>1\end{cases}$$

$\alpha>0 \quad \int_1^{+\infty}\frac{dx}{x^\alpha} = \lim_{R\to+\infty}\int_1^R \frac{dx}{x^\alpha}$

$$\int_1^R \frac{dx}{x^\alpha} = \left[\frac{x^{1-\alpha}}{1-\alpha}\right]_1^R = \begin{cases} \dfrac{R^{1-\alpha}}{1-\alpha} - \dfrac{1}{1-\alpha}, & 0<\alpha<1 \\[2ex] -\dfrac{1}{\alpha-1}\dfrac{1}{R^{\alpha-1}}+\dfrac{1}{\alpha-1}, & \alpha>1\end{cases}$$

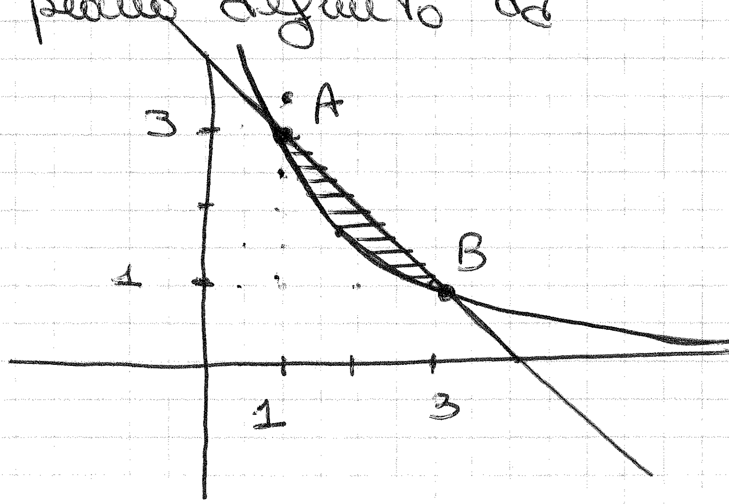
$\alpha \neq -1$

$$\int_1^R \frac{dx}{x} = \left[\log|x|\right]_1^R = \log R$$

$$\int_1^{\infty}\frac{dx}{x^\alpha} = \begin{cases} \lim\limits_{R\to+\infty}\left(\dfrac{R^{1-\alpha}}{1-\alpha} - \dfrac{1}{1-\alpha}\right) = +\infty, & 0<\alpha<1 \\[2ex] \lim\limits_{R+\infty} \log R = +\infty, & \alpha = 1 \\[2ex] \lim\limits_{R\to+\infty}\left(-\dfrac{1}{\alpha-1}\dfrac{1}{R^{\alpha-1}}+\dfrac{1}{\alpha-1}\right) = \dfrac{1}{\alpha-1}, & \alpha>1\end{cases}$$

Calcolare l'area del dominio piano definito da

$$\begin{cases} xy \geq 3 \\ x+y \leq 4 \\ x \geq 0 \end{cases} \qquad y=\frac{3}{x} \qquad y = 4-x$$



$$\begin{cases} xy=3 \\ x+y=4 \end{cases} \quad A\begin{cases} x_1 = 1 \\ y_1 = 3\end{cases} \quad B\begin{cases} x_2 = 3 \\ y_1 = 1\end{cases}$$

$$A = \int_1^3 (4-x)\,dx - \int_1^3 \frac{3}{x}\,dx$$

$$= \int_1^3 \left(4-x-\frac{3}{x}\right)dx = \left[4x - \frac{x^2}{2} - 3\log|x|\right]_1^3 =$$

$$= \left(12 - \frac{9}{2} - 3\log 3\right) - \left(4 - \frac{1}{2}\right) = 4 - 3\log 3$$