CORSO DI ANALISI MATEMATICA I A

CORSO DI LAUREA TRIENNALE IN FISICA

Foglio di esercizi numero 11

11.1 Verificare che $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$ converge ed ha per somma $1/2$.

11.2 Dimostrare per induzione

$$\sum_{n=1}^{k} \ln\left[\frac{(n+1)^2}{n(n+2)}\right] = \ln \frac{2(k+1)}{k+2}$$

e calcolare la somma della serie

$$\sum_{n=1}^{\infty} \ln\left[\frac{(n+1)^2}{n(n+2)}\right].$$

11.3 Dimostrare per induzione

$$\sum_{n=1}^{k} \frac{2}{n(n+2)} = \frac{3}{2} - \frac{1}{k+1} - \frac{1}{k+2}$$

e calcolare la somma della serie

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}.$$

11.4 Studiare la convergenza delle serie seguenti:

(1) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{n}$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

(4) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$

(6) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{3n-1}\right)^n$

(7) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

(9) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n} - \operatorname{sen} \frac{1}{n}\right)$

(10) $\displaystyle\sum_{n=1}^{\infty} \left(e^{1/n} - e^{-1/n}\right)$

(11) $\displaystyle\sum_{n=1}^{\infty} \left(1 - n^2 \operatorname{sen}^2 \frac{1}{n}\right)$

(12) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

(13) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

(14) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

11.5 Studiare la convergenza delle serie seguenti:

(1) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)$

(2) $\displaystyle\sum_{n=1}^{\infty}\left(3^{\frac{1}{2n+1}}-3^{\frac{1}{2n-1}}\right)$

(3) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n-\sqrt{n}}$

(4) $\displaystyle\sum_{n=1}^{\infty}n^{-\ln(1+x)}$

(5) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln^2 n}{n\sqrt{n}}$

(6) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2-n\,\text{sen}^2(n)}$

(7) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n+5^n}{n!}$

(8) $\displaystyle\sum_{n=1}^{\infty}\frac{n\cos(n\pi)}{1+n^2}$

(9) $\displaystyle\sum_{n=1}^{\infty}\frac{1+n}{1+n^2}$

(10) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{n^2}}{n^{2n}}$

(11) $\displaystyle\sum_{n=1}^{\infty}\frac{1+\cos n}{1-\text{sen}\,n}$

(12) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n!}$

(13) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^2}$

(14) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n-\sqrt{n}}$

(15) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+\text{sen}^2 n}$

(16) $\displaystyle\sum_{n=1}^{\infty}\frac{1+e^n}{n!}$

(17) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n^n}$

(18) $\displaystyle\sum_{n=1}^{\infty}\frac{\text{tg}(\text{sen}\,n)}{n^2}$

(19) $\displaystyle\sum_{n=1}^{\infty}n^2(1+\text{sen}\,n)$

(20) $\displaystyle\sum_{n=1}^{\infty}\left(\cos\frac{1}{n}-1\right)$

(21) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{n^2}}{n^{2n}}$

(22) $\displaystyle\sum_{n=1}^{\infty}\frac{\text{sen}(1/n)}{n}$

(23) $\displaystyle\sum_{n=1}^{\infty}\frac{n+n^2}{n!}$

(24) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^2+2^n}$