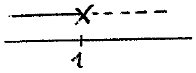
Soluzioni [1]

1\. C.E. $\mathbb{R} - \{1\}$

SGN



LIM per $x \longrightarrow 1^+$ $f(x) \longrightarrow -\infty$ as. verticale

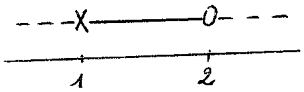
per $x \to 1^-$ $f(x) \longrightarrow 0$ d.e. da sinistra

per $x \longrightarrow \pm\infty$ $f(x) \sim -\frac{x}{e} \to \mp\infty$

$$f(x) + \frac{x}{e} = (1-x)e^{\frac{2-x}{x-1}} + x e^{-1} \underset{x=\frac{1}{t}\to 0^\pm}{=} \frac{(t-1)e^{\frac{t}{1-t}}+1}{et} \underset{H}{=} \dots \to 0^\mp$$

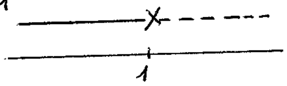
$y = -\frac{x}{e}$ asintoto obliquo

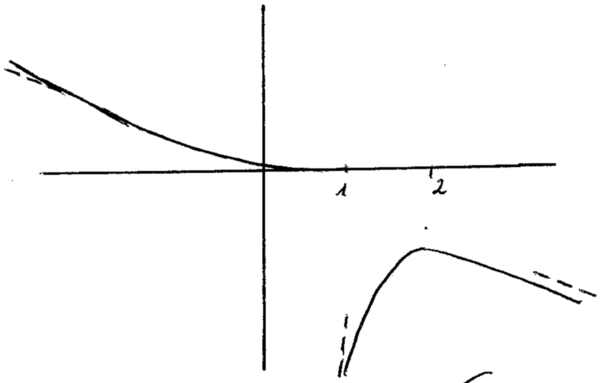
DRV $f'(x) = \frac{2-x}{x-1} e^{\frac{2-x}{x-1}}$



per $x \to 1^-$ $f'(x) \sim \frac{1}{x-1} e^{\frac{1}{x-1}} \longrightarrow 0$

DRV² $f''(x) = -\frac{1}{(x-1)^3} e^{\frac{2-x}{x-1}}$





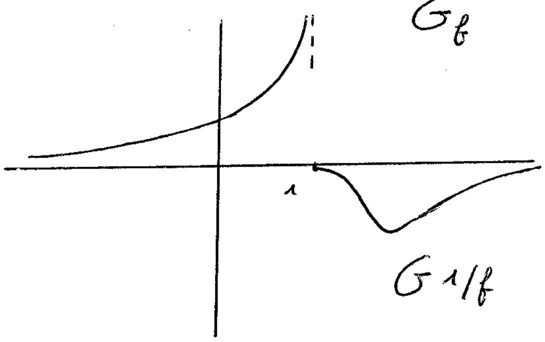
G_f

$g = \frac{1}{f}$

$g' = -f'/f^2 = \frac{2-x}{(1-x)^3} e^{-\frac{2-x}{x-1}}$

$g'' = \frac{2f'^2 - ff''}{f^3} = \frac{2x^2-8x+7}{(1-x)^5} e^{\frac{2-x}{x-1}}$

$x = 2 \pm \sqrt{2}/2$ punti di flesso



$G_{1/f}$

2\. $\operatorname{sen} x = x + o(x^2)$

$\lg(1+\operatorname{sen} x) = \lg(1+x+o(x^2)) = x - \frac{x^2}{2} + o(x^2)$

$\operatorname{sen}(x^2) = x^2 + o(x^4)$

$\sqrt{1-\operatorname{sen}(x^2)} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + o(x^4)$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$

$$f(x) \sim \frac{-\frac{1}{2}x^2}{-\frac{1}{6}x^4} \longrightarrow +\infty$$

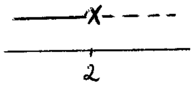
3\. Si deve applicare il teorema di Lagrange alla funzione $f(x) = e^{-x}$ nell'intervallo $[1, e]$, dopo aver osservato che le ipotesi del teorema sono soddisfatte. L'ascissa x del punto P è soluzione dell'equazione

$$f'(x) = -\frac{1}{2\lg 2} \iff -e^{-x} = -\frac{1}{2\lg 2} \iff x = \lg(2\lg 2) \in (0, \lg 2)$$

Soluzioni [2]

1\. C.E. $\mathbb{R} - \{2\}$

SGN



LIM per $x \to 2^+$ $f(x) \to 0$ d.e. da destra

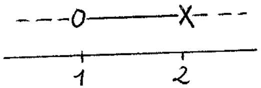
per $x \to 2^-$ $f(x) \to +\infty$ as. verticale

per $x \to \pm\infty$ $f(x) \sim -ex$

$$f(x) + ex = (2-x)e^{\frac{x-3}{x-2}} + ex \underset{x=\frac{1}{t} \to 0^\pm}{=} e^{\frac{1-3t}{1-2t}} \frac{4t-3}{2t-1} \to 3e$$

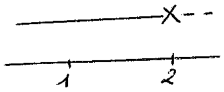
$y = 3e - ex$ asintoto obliquo

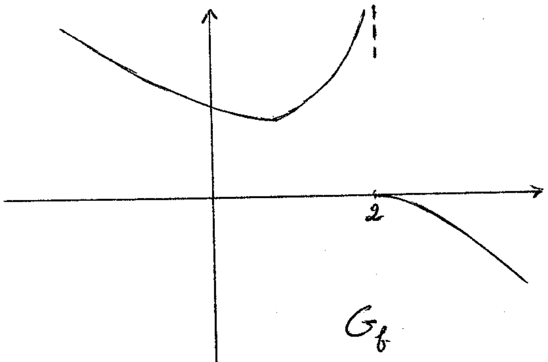
DRV $f'(x) = \dfrac{1-x}{x-2} e^{\frac{x-3}{x-2}}$



per $x \to 2^+$ $f'(x) \sim -\dfrac{e^{-\frac{1}{x-2}}}{x-2} \to 0$

DRV^2 $f''(x) = \dfrac{1}{(2-x)^3} e^{\frac{x-3}{x-2}}$



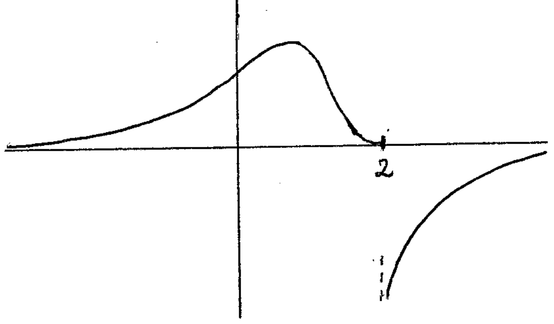


$g = 1/f$

$g' = -f'/f^2 = \dfrac{1-x}{(2-x)^3} e^{-\frac{x-3}{x-2}}$

$g'' = \dfrac{2f'^2 - ff''}{f^3} = \dfrac{2x^2-4x+1}{(2-x)^5} e^{-\frac{x-3}{x-2}}$

$x = 1 \pm 1/\sqrt{2}$ punti di flesso



2\. $\cos x^2 = 1 - \frac{1}{2}x^4 + o(x^4)$

$\sqrt{\cos x^2} = 1 - \frac{1}{4}x^4 + o(x^4)$

$(\operatorname{sen} x)^2 = x^2 - \frac{1}{3}x^4 + o(x^4)$

$\operatorname{tg} x = x + \frac{x^3}{3} + o(x^4)$

$\lg(1 + \operatorname{tg} x) = x - \frac{1}{2}x^2 + o(x^2)$

$$f(x) \sim \frac{\frac{1}{12}x^4}{-\frac{3}{2}x^2} \to 0$$

3\. Si deve applicare il teorema di Lagrange alla funzione $f(x) = \lg x^2$ nell'intervallo $[1, e]$, dopo aver osservato che le ipotesi del teorema sono soddisfatte. L'ascissa del punto P è soluzione dell'equazione

$$f'(x) = \frac{2}{e-1} \iff \frac{2}{x} = \frac{2}{e-1} \iff x = e-1 \in (1, e).$$

Soluzioni [3]

1\. C.E. $\mathbb{R} - \{1\}$

SGN - - - x ———

1

LIM per $x \to 1^+$ $f(x) \to 0$ d.e. da destra

per $x \to 1^-$ $f(x) \to -\infty$ asintoto verticale

per $x \to \pm\infty$ $f(x) \sim ex \to \pm\infty$

$$f(x) - ex \underset{x=\frac{1}{t}\to 0^\pm}{=} \frac{(1-t)e^{\frac{3t-1}{t-1}} - e}{t} \underset{H.}{=} \dots \to -3e$$

$y = ex - 3e$ asintoto obliquo

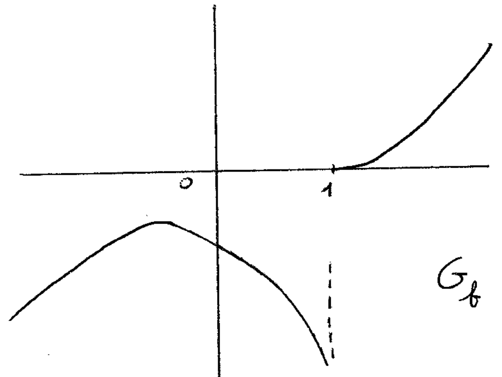
DRV $f'(x) = \dfrac{x+1}{x-1} e^{\frac{3-x}{1-x}}$ ——0- - - -x———

-1 1

per $x \to 1^+$ $f'(x) \sim \dfrac{2}{x-1} e^{\frac{2}{1-x}} \longrightarrow 0$

DRV^2 $f''(x) = \dfrac{4}{(x-1)^3} e^{\frac{3-x}{1-x}}$ - - - - - - x———

1

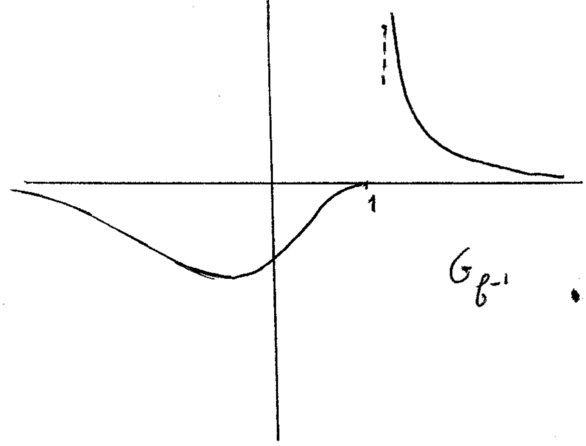


$g = 1/f$

$g' = -f'/f^2 = \dfrac{1+x}{(1-x)^3} e^{-\frac{3-x}{1-x}}$

$g'' = \dfrac{2f'^2 - ff''}{f^3} = \dfrac{2x^2+4x-2}{(x-1)^2} e^{-\frac{3-x}{1-x}}$

$x = -1 \pm \sqrt{2}$ punti di flesso



2\. $\operatorname{sen} x^2 = x^2 - \frac{1}{3}x^4 + o(x^4)$

$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2)$

$\sqrt{1+\operatorname{sen} x^2} = 1 + \frac{1}{2}x^2 - \frac{7}{24}x^4 + o(x^4)$

$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$

$\sqrt{1-x^2} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + o(x^4)$

$$f(x) \sim \frac{-\frac{1}{4}x^4}{-\frac{1}{8}x^4} \to 2$$

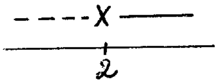
3\. Si deve applicare il teorema di Lagrange alla funzione $f(x) = 2^x$ nell'intervallo $[0,1]$, dopo aver osservato che le ipotesi del teorema sono soddisfatte. L'ascissa x del punto P è soluzione dell'equazione:

$$f'(x) = 1 \iff 2^x \lg 2 = 1 \iff x = -\frac{\lg \lg 2}{\lg 2} \in (0,1).$$

Soluzioni [4]

1. C.E. $\mathbb{R} - \{2\}$

SGN



LIM per $x \to 2^+$ $f(x) \to +\infty$ as. verticale

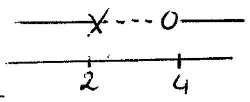
per $x \to 2^-$ $f(x) \to 0$ d.e. da sinistra

per $x \to \pm\infty$ $f(x) \sim e^{-1}x \to \pm\infty$

$$f(x) - e^{-1}x \underset{x=\frac{1}{t}\to 0^{\pm}}{=} \frac{(1-2t)\,e^{\frac{1-4t}{2t-1}} - e^{-1}}{t} \underset{H}{=} \cdots \to 0$$

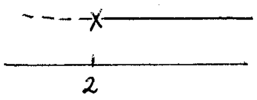
$y = e^{-1}x$ asintoto obliquo

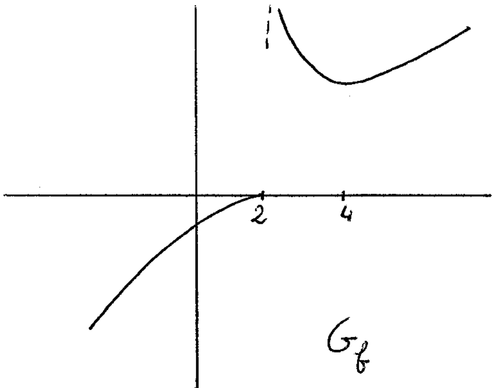
DRV $f'(x) = \dfrac{x-4}{x-2}\, e^{\frac{x-4}{2-x}}$



per $x \to 2^-$ $f'(x) \sim \dfrac{2}{2-x}\, e^{\frac{2}{2-x}} \to 0$

DRV^2 $f''(x) = \dfrac{4}{(x-2)^3}\, e^{\frac{x-4}{2-x}}$



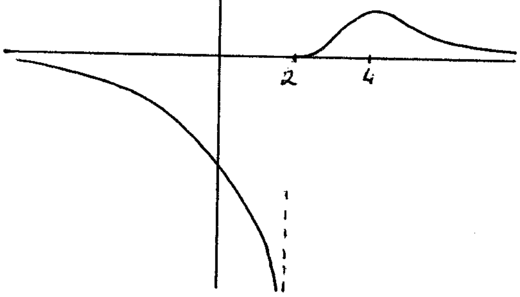


$g = 1/f$

$g' = -f'/f^2 = \dfrac{x-4}{(2-x)^3}\, e^{-\frac{x-4}{2-x}}$

$g'' = \dfrac{2f'^2 - ff''}{f^3} = \dfrac{2x^2-16x+28}{(x-2)^5}\, e^{-\frac{x-4}{2-x}}$

$x = 4 \pm \sqrt{2}$ punti di flesso



2.

$\lg(1+x^2) = x^2 - \frac{1}{2}x^4 + o(x^4)$

$\sqrt{1+x^2} = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + o(x^4)$

$\cos\sqrt{x} = 1 - \frac{1}{2}x + \frac{1}{24}x^2 + o(x^2)$

$\cos^2\sqrt{x} = 1 - x + \frac{1}{3}x^2 + o(x^2)$

$\cos^2 x = 1 - x^2 + o(x^2)$

$$f(x) \sim \frac{-\frac{1}{4}x^4}{\frac{4}{3}x^4} \to -\frac{3}{16}$$

3. Si deve applicare il teorema di Lagrange alla funzione $f(x) = \lg\frac{1}{x}$ nell'intervallo $[1, e]$, dopo aver osservato che le ipotesi del teorema sono soddisfatte: l'ascissa x del punto P è soluzione dell'equazione:

$$f'(x) = \frac{1}{1-e} \iff -\frac{1}{x} = \frac{1}{1-e} \iff x = e-1 \in (1, e)$$