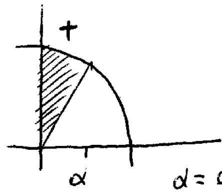
Soluzioni

1. Periodo 2π $\rightarrow$ si studia in $[-\pi, \pi]$
Dispari $\rightarrow$ si studia in $[0, \pi]$
Simmetria rispetto alla retta $x = \pi/2$ $\rightarrow$ si studia in $[0, \frac{\pi}{2}]$

Dom$f = [0, \pi/2)$

LIM. per $x \to \frac{\pi}{2}$ $f(x) \to +\infty$ (as. verticale) ; $f(0) = 0$



$\alpha = \arccos\sqrt{\frac{2}{3}}$

DRV $f'(x) = \dfrac{3\cos^4 x - 5\cos^2 x + 2}{\cos x}$

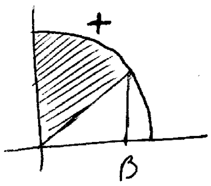
$3\cos^4 x - 5\cos^2 x + 2 \geq 0 \iff \cos^2 x \geq 1 \vee \cos^2 x \leq \frac{2}{3}$

DRV^2 $f''(x) = \dfrac{\operatorname{sen} x\,(-9\cos^4 x + 5\cos^2 x + 2)}{\cos^2 x}$

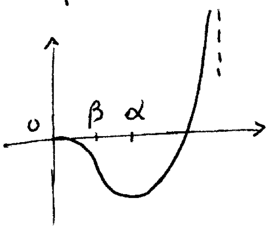
$-9\cos^4 x + 5\cos^2 x + 2 \geq 0 \iff \frac{5 - \sqrt{97}}{18} \leq \cos^2 x \leq \frac{5 + \sqrt{97}}{2}$

$\iff 0 \leq \cos x \leq \sqrt{\frac{5 + \sqrt{97}}{2}} = \rho$

$\iff \arccos \rho \leq x \leq \pi/2$



$\beta = \arccos \rho$



$f(x) = \lg\left(1 + x - \frac{x^3}{6} + o(x^4)\right) - \lg\left(1 - x + \frac{x^3}{6} + o(x^4)\right) - 2\left(x - \frac{x^3}{6} + o(x^4)\right) - \left(x - \frac{x^3}{6} + o(x^4)\right)^3$

$= \left[\left(x - \frac{x^3}{6}\right) - \frac{1}{2}x^2 + \frac{1}{3}x^3\right] - \left[\left(-x + \frac{x^3}{6}\right) - \frac{1}{2}x^2 - \frac{1}{3}x^3\right] - 2x + \frac{x^3}{3} - x^3 + o(x^3)$

$= -\frac{1}{3}x^3 + o(x^3)$.

2. Per $x \to 0^+$ $\dfrac{\operatorname{sen} x}{\lg(1+x)} \sim \dfrac{x}{x} \to 1$; per $x \to 0^-$ $e^{\frac{\lg \cos x}{x}} \sim e^{\frac{-x^2/2}{x}} \to 1$

Si può prolungare per continuità ponendo $f(0) = 1$.

$\lim_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \dfrac{\frac{\operatorname{sen} x}{\lg(1+x)} - 1}{x} \sim \dfrac{\operatorname{sen} x - \lg(1+x)}{x \lg(1+x)} \sim \dfrac{+x^2/2}{x^2} \to +\frac{1}{2}$

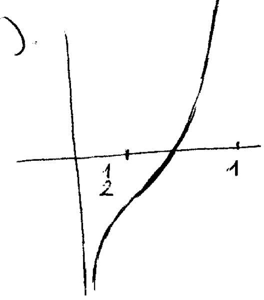
$\lim_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \dfrac{e^{\frac{\lg \cos x}{x}} - 1}{x} \sim \dfrac{\lg \cos x}{x^2} \sim \dfrac{-1/2\, x^2}{x^2} \to -\frac{1}{2}$

$f'(0)$ non esiste (punto angoloso).

3. $f(x) = \lg x + 2x^2 - x$, $x > 0$

$f'(x) = \frac{1}{x} + 4x - 1 = \dfrac{4x^2 - x + 1}{x} > 0$

$f''(x) = 4 - \frac{1}{x^2} > 0$ per $x > \frac{1}{2}$

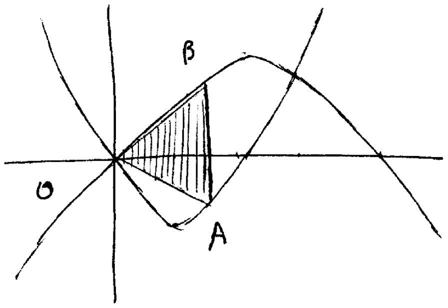


nell'intervallo $[\frac{1}{2}, 1]$ possiamo applicare il metodo delle tangenti:

$$\begin{cases} b_0 = 1 \\ b_{n+1} = b_n - \dfrac{\lg b_n + 2b_n^2 - b_n}{\frac{1}{b_n} + 4b_n - 1} \end{cases}$$

$\rightarrow b_2 = 0{,}75$ $\quad b_3 = 0{,}72\ldots$

4.



$A = (x,\ x^2 - 2x) \qquad 0 \le x \le 4$

$B = (x,\ 4x - x^2)$

$\mathcal{A} = \frac{1}{2} x \left(4x - x^2 - x^2 + 2x\right) = (3 - x)\, x^2$

max per $x = 2$

5. f, g funzioni continue in $[a,b]$, derivabili in (a,b), con $g' \neq 0$ in (a,b)

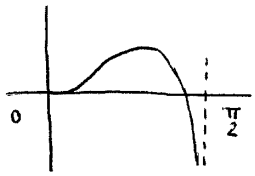
$\Downarrow$

$$\exists\, \xi \in (a,b) : \frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(\xi)}{g'(\xi)}$$

Le funzioni date verificano le ipotesi del teorema.
Cerchiamo le soluzioni dell'equazione $e^x = \frac{e^2 - 1}{2}$; dunque $\xi = \lg \frac{e^2 - 1}{2}$.

Soluzioni [2]

1. La funzione è l'opposta di quella studiata in [1].
La soluzione del problema si ricava immediatamente da quella fornita.



$$f(x) = \frac{1}{3}x^3 + o(x^3)$$

2. Per $x \to 0^+$ $f(x) \sim \frac{x}{x} \to 1$; per $x \to 0^-$ $e^{\frac{\lg \cos x}{x}} \sim e^{\frac{-x^2/2}{x}} \to 1$
si può prolungare per continuità ponendo $f(0) = 1$.

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\frac{\lg(1+x)}{\operatorname{sen} x} - 1}{x} = \lim_{x\to 0} \frac{\lg(1+x) - \operatorname{sen} x}{x \operatorname{sen} x} \sim \frac{-x^2/2}{x^2} \to -\frac{1}{2}$$

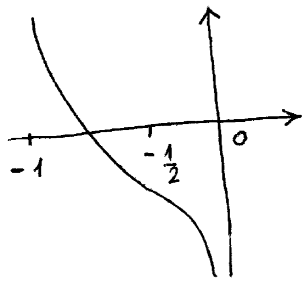
$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{e^{\frac{\lg\cos x}{x}} - 1}{x} \sim \frac{\lg \cos x}{x^2} \sim \frac{-\frac{1}{2}x^2}{x^2} \to -\frac{1}{2}$$

$f'(0)$ esiste e vale $-1/2$.

3. $f(x) = \lg(-x) + 2x^2 + x \quad , \quad x < 0$

$f'(x) = \frac{1}{x} + 4x + 1 = \frac{4x^2+x+1}{x} > 0$

$f''(x) = 4 - \frac{1}{x^2} > 0$ per $x < -\frac{1}{2}$

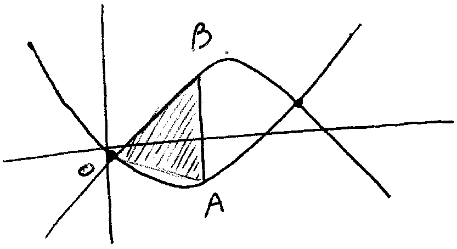


nell'intervallo $[-1, -\frac{1}{2}]$ possiamo applicare il metodo delle tangenti.

$$\begin{cases} b_0 = -1 \\ b_{n+1} = b_n - \dfrac{\lg(-b_n) + 2b_n^2 + b_n}{\frac{1}{b_n} + 4b_n + 1} \end{cases}$$

$\to b_2 = -0,75 \qquad b_3 = -0,72\ldots$

4.



$A = (x, \frac{x^2}{4} - x)$

$B = (x, 2x - \frac{x^2}{4}) \qquad 0 \le x \le 6$

$$\mathcal{A} = \frac{1}{2}x\left(2x - \frac{x^2}{4} - \frac{x^2}{4} + x\right) = \frac{6x^2 - x^3}{4}$$

max per $x = 4$.

5. f, g funzioni continue in $[a,b]$, derivabili in (a,b), $g' \neq 0$ in (a,b).

$\Downarrow$

$$\exists\, \xi \in (a,b) : \frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(\xi)}{g'(\xi)}$$

Poiché $g'(x) = 2x$ si annulla per $x = 0 \in (-1,3)$, le ipotesi del teorema non sono verificate.
Controlliamo se vale la tesi:

$$\frac{4x^3 - 12x^2}{2x} = -\frac{32}{8} \longleftrightarrow x^2 - 3x + 2 = 0 \longleftrightarrow x = 1,\ x = 2$$

entrambe accettabili.