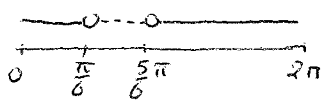
Soluzioni

1.

CE Deve essere $\left|\dfrac{2\,\mathrm{sen}\,x-1}{\mathrm{sen}\,x-2}\right| \le 1 \underset{\text{elevando al quadrato}}{\Longleftrightarrow} \mathrm{sen}^2 x \le 1$, che è sempre verificata.

Dunque $CE = \mathbb{R}$. Essendo però la fz. 2π-periodica, basta studiarla per $x \in [0, 2\pi]$.

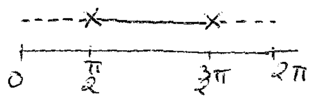
SEN $f(x) \ge 0 \iff \dfrac{2\,\mathrm{sen}\,x-1}{\mathrm{sen}\,x-2} \ge 0 \iff \mathrm{sen}\,x \le \frac{1}{2}$



$f(0) = f(\pi) = f(2\pi) = \frac{\pi}{6}$

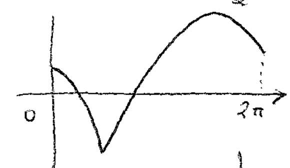
$f(\frac{\pi}{2}) = -\frac{\pi}{2}$, $f(\frac{3}{2}\pi) = \frac{\pi}{2}$

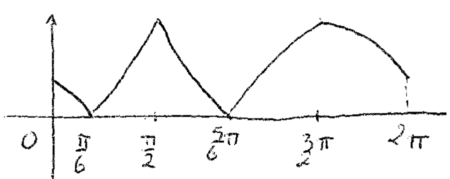
DRV $f'(x) = \dfrac{\sqrt{3}\,\mathrm{sgn}\cos x}{\mathrm{sen}\,x-2}$



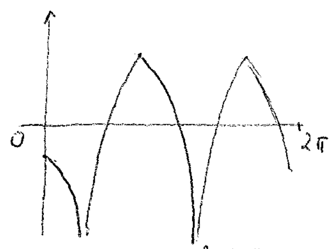
$x = \frac{\pi}{2}$, $x = \frac{3}{2}\pi$ punti angolosi

$f''(x) = \dfrac{-\sqrt{3}\,|\cos x|}{(\mathrm{sen}\,x-2)^2}$ la fz è concava

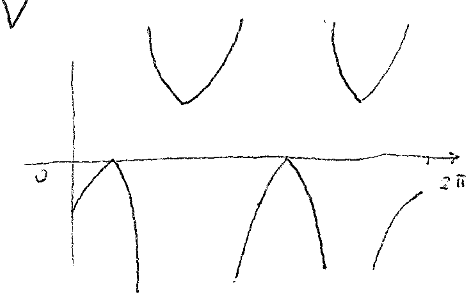




$y = |f|$



$y = \lg|f|$



2. $\displaystyle\int_1^{+\infty} \frac{\lg(1+x)}{x^2}\,dx$ per $x \to +\infty$ $\; f(x) < \dfrac{x^\alpha}{x^2} = \left(\dfrac{1}{x}\right)^{2-\alpha}$. Scelto α t.c. $2-\alpha > 1$ ....

$\displaystyle\int_0^{1} \frac{\lg(1+x)}{x^2}\,dx$ per $x \to 0$ $\; f(x) \sim \dfrac{x}{x^2} = \dfrac{1}{x}$ ....

$$\int_1^{+\infty} \frac{\lg(1+x)}{x^2}\,dx = \left[-\frac{1}{x}\lg(1+x)\right]_1^{+\infty} + \int_1^{+\infty} \frac{dx}{x(x+1)} = \lg 2 + \int_1^{+\infty}\left(\frac{1}{x} - \frac{1}{x+1}\right)dx =$$

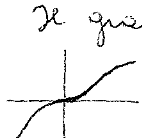
$$= \lg 2 + \left[\lg\frac{x}{x+1}\right]_1^{+\infty} = \lg 2 - \lg\frac{1}{2} = 2\lg 2.$$

Volume $= 2\pi \lg 2$.

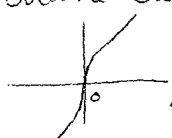
3. $yy' = x \implies \int y\,dy = \int x\,dx \implies \dfrac{y^2}{2} = \dfrac{x^2}{2} + \dfrac{c}{2} \implies y = \pm\sqrt{x^2+c}$.

Tenendo conto del dato iniziale, ci interessa la funzione positiva, con $c = 4$: $y(x) = \sqrt{x^2+4}$.

4. Poiché $f'(x) = 4x^2(x^2+3)/(x^2+1)^2 > 0$, la funzione è crescente e dunque invertibile.
Il grafico di f^{-1} si ottiene da quello di $f(x)$ per simmetria rispetto a $y = x$.



$y = f(x)$



$y = f^{-1}(x)$

Si crea un punto a tangente verticale nell'origine.

5. $x_n = \lg \dfrac{4}{\sqrt{n^2+3} + \sqrt{n^2+1}} \to -\infty$

Dobbiamo provare che, fissato $M > 0$, definitivamente è $\lg(\sqrt{n^2+3} - \sqrt{n^2-1}) < -M \iff$
$\iff \sqrt{n^2+3} < e^{-M} + \sqrt{n^2-1} \iff 2e^{-M}\sqrt{n^2-1} > 4 - e^{-2M} \iff \sqrt{n^2-1} > \dfrac{4e^M - 1}{2} \iff \dots$