18/5/2018

Recupero 22/5 alle 16.30 in B22

$$x^2+5y^2+(k^2+9)z^2+2xy-2kxz+2kyz-1=0$$ 13.4.1 (n)

$$\begin{pmatrix} 1 & 1 & -k & 0 \\ 1 & 5 & k & 0 \\ -k & k & k^2+9 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$d_1 = 1$

$d_2 = 4$

$d_3 =$

$$= \det\begin{pmatrix} 5 & k \\ k & k^2+9 \end{pmatrix} - \det\begin{pmatrix} 1 & -k \\ k & k^2+9 \end{pmatrix} + k \det\begin{pmatrix} 1 & k \\ 5 & -k \end{pmatrix} =$$

$$= 3k^2 + 45 - k^2 - k^2 - 9 - k^2 + k^2(-6) =$$

$$= -4k^2 + 36 = 4(9 - k^2)$$

$$d_4 = -d_3$$

Quadrica degenere $\iff d_4 = 0 \iff k = \pm 3$

Se $k \in (-3, 3)$: $d_3 > 0$, $d_4 < 0$

Ho $d_1, d_2, d_3 > 0$, $d_4 < 0 \implies$ ellissoide

Se $k \in (-\infty, -3) \cup (3, +\infty)$: $d_1, d_2 > 0$, $d_3 < 0$

$\Rightarrow$ conica all'∞ è non degenere. $\quad d_4 > 0$

Chiusura proiettiva: iperboloide.

$\Rightarrow$ iperboloide iperbolico (a una falda).

13.4.1 (a) $\quad x^2 + (k^2+1)y^2 + (k^2+2)z^2 - 2kxy + 2xz - 4kyz - 1 = 0$

$$\begin{pmatrix} 1 & -k & 1 & 0 \\ -k & k^2+1 & -2k & 0 \\ 1 & -2k & k^2+2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$d_1 = 1$, $\quad d_2 = k^2+1-k^2 = 1$

$$d_3 = (k^2+1)(k^2+2) - 4k^2 + k\left(-k(k^2+2) + 2k\right)$$

$$+ (2k^2 - k^2 - 1) = 1$$

$$d_4 = -d_3 = -1 .$$

Quadrica è non degenere $\forall k$.

Ellissoide.

13.4.1 (P): $x^2 + (k^2-1)y^2 - k^2 z^2 + 2kxy - 2kyz - 4x - 4ky$

$- 2z + 4 = 0$

$$\begin{pmatrix} 1 & k & 0 & -2 \\ k & k^2-1 & -k & -2k \\ 0 & -k & -k^2 & -1 \\ -2 & -2k & -1 & 4 \end{pmatrix}$$

$d_1 = 1$, $d_2 = k^2 - 1 - k^2 = -1$

$$d_3 = -k^2(k^2-1) - k^2 - k(-k^3) =$$
$$= -k^4 + k^2 - k^2 + k^4 = 0$$

Conica all'∞: coppia di rette.

2 possibilità:
- $d_4 = 0 \Rightarrow$ quadrica è degenere
- $d_4 \neq 0 \Rightarrow$ paraboloide iperbolico.

x casa: calcolare d_4 e scoprire la risposta.

Curve:

14.1.2:

$$\alpha : [0,1] \longrightarrow \mathbb{R}^2$$
$$t \longrightarrow (t, 1-t^2)$$

$$\beta : [0, \tfrac{\pi}{2}] \longrightarrow \mathbb{R}^2$$
$$t \longrightarrow (\sin t, \cos^2 t)$$

$$\exists \ \tau : [0, \tfrac{\pi}{2}] \longrightarrow [0,1] \ : \ \beta(t) = \alpha(\tau(t)) \ ?$$

biiettiva e continua

Scelgo $\tau(t) = \sin t$: $\beta(\tau(t)) = (\tau(t), 1-\tau(t)^2)$

$$= (\sin t,\ 1-(\sin t)^2) = (\sin t,\ (\cos t)^2)$$

Sì!

14.1.3

$$\alpha, \beta : [0,1] \longrightarrow \mathbb{R}^2$$

$$\alpha(t) = (2t^2,\ 3t^3),\quad \beta(t) = (3t^2,\ 2t^3)$$

confrontare le lunghezze?

$$\alpha'(t) = (4t,\ 9t^2), \qquad \beta'(t) = (6t,\ 6t^2)$$

$$\ell(\alpha) = \int_0^1 \|\alpha'(t)\|\,dt = \int_0^1 (16t^2 + 81t^4)^{\frac{1}{2}}\,dt =$$

$$= \int_0^1 t\,(16 + 81t^2)^{\frac{1}{2}}\,dt$$

$$\ell(\beta) = \int_0^1 \|\beta'(t)\|\,dt = \int_0^1 6t\,(1 + t^2)^{\frac{1}{2}}\,dt$$

$t > 0$

$$\|\alpha'(t)\| > \|\beta'(t)\| \iff$$

$$\iff \cancel{t}\,(16 + 81t^2)^{\frac{1}{2}} > 6\cancel{t}\,(1 + t^2)^{\frac{1}{2}}$$

$$\iff (16 + 81t^2)^{\frac{1}{2}} > 6\,(1 + t^2)^{\frac{1}{2}}$$

$$\Longleftrightarrow \quad 16 + 81 t^2 > 6(1+t^2)$$ $\checkmark$ per $t>0$,

$$\Rightarrow \quad \ell(\alpha) > \ell(\beta).$$

In alternativa, i 2 integrali possono essere calcolati esplicitamente con la sost. $y = t^2$.

$dy = 2t\,dt$

$$\ell(\beta) = \int_0^1 6t(1+t^2)^{\frac{1}{2}}\,dt = \int_0^1 3\sqrt{1+y}\,dy =$$

14.1.3 (a)

$\alpha : [0, 2\pi] \longrightarrow \mathbb{R}^3$, $\alpha(t) = (\sin t, \cos t, t^2)$

$\beta : [0, \pi] \longrightarrow \mathbb{R}^3$, $\beta(t) = (\sin(2t), \cos(2t), 4t^2)$

confrontare $\ell(\alpha)$ e $\ell(\beta)$:

Osservo che: $\beta = \alpha(2t)$

β è ottenuta da α tramite il cambio di parametro $\tau : [0, \pi] \longrightarrow [0, 2\pi]$

$t \longrightarrow 2t$

continuo biyettivo, $\tau' \neq 0$

$\Rightarrow \ell(\alpha) = \ell(\beta)$

$$\alpha'(t) = (\cos t, -\sin t, 2t) \qquad \|\alpha'(t)\| = (1+4t^2)^{\frac{1}{2}}$$

$$\ell(\alpha) = \int_0^{2\pi} (1+4t^2)^{\frac{1}{2}}\, dt$$

$$\beta'(t) = (2\cos 2t, -2\sin 2t, 8t) \qquad \|\beta'(t)\| = 2\sqrt{1+16t^2}$$

$$\ell(\beta) = \int_0^{\pi} 2\sqrt{1+16t^2}\, dt = \qquad 2t = y, \quad 2\,dt = dy$$

$$= \int_0^{2\pi} \sqrt{1+4y^2}\, dy = \ell(\alpha),$$