13/3/2015

ESERCITAZIONI

ES. 9 (FOGLIO 9/3/2015)

(b) $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ $\langle \cdot | \cdot \rangle_A$ è ovviamente bilineare, simmetrico

guardo i minori $A_{1;1} = (1)$

$A_{1,2;1,2} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$

$A_{1,2,3;1,2,3} = A$

$\det A_1 = 1 > 0$

$\det A_{1,2;1,2} = 1 > 0$

$\det A = 4 > 0$ $\Rightarrow \langle \cdot | \cdot \rangle_A$ è def. > 0

$\Rightarrow \langle \cdot | \cdot \rangle_A$ è p. scalare

(c) $A = \begin{pmatrix} 7 & 2 & 0 \\ 0 & 5 & 3 \\ 4 & 0 & 9 \end{pmatrix}$

$A_{1;2} \neq A_{2;1}$ $\Rightarrow \langle \cdot | \cdot \rangle_A$ non è simmetrico

$\Rightarrow \langle \cdot | \cdot \rangle$ non è p. scalare.

13/3/2015

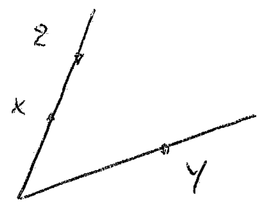
ESERCITAZIONI

ES. 14 (FOGLIO 9/3/2015)

$SNCF(x,y) = d(x,y)$

- $d(x,y) > 0$ se $x \neq y$ ovvio
- $d(x,y) = d(y,x)$ ovvio
- $d(x,y) \leq d(x,z) + d(z,y)$

a) se z dipendente da x o da y (suppongo da x) e se y indipendente da x (altrimenti ho la usuale disuguaglianza triangolare su $\mathbb{R}$)

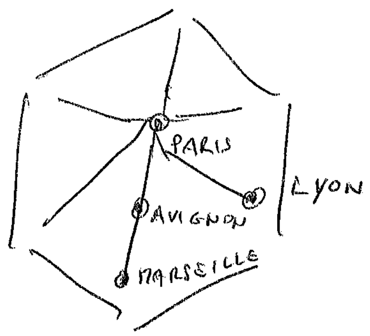


$$d(x,y) = \|x\| + \|y\| \leq \|x-z\| + \|z\| + \|y\| = d(x,z) + d(z,y)$$

b) se z indipendente da x e da y

$$d(x,y) = \begin{cases} \|x\| + \|y\| \\ \|x-y\| \end{cases} \leq \|x\| + \|y\| \leq \|x\| + \|z\| + \|z\| + \|y\| = d(x,z) + d(y,z)$$

c) il caso x, y, z dipendenti è banale: si riduce alla usuale disuguaglianza triangolare per la distanza standard in $\mathbb{R}$.



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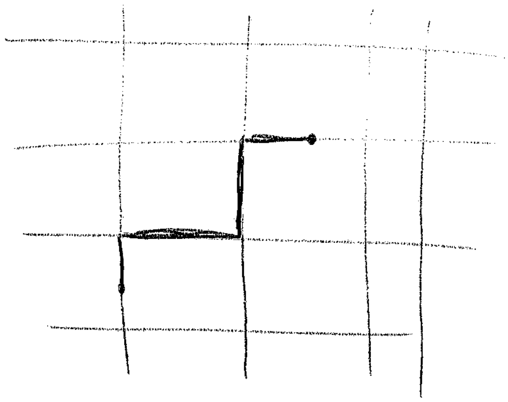
$d(x,y) = NYC(x,y)$

- se $x \neq y$ $\quad d(x,y) \neq 0$ ovvio perchè almeno una delle coordinate è diversa
- $d(x,y) = d(y,x)$ ovvio
- $d(x,y) = \sum |x_i - y_i| \leq \sum \left(|x_i - z_i| + |z_i - y_i| \right) = d(x,z) + d(z,y)$



New York City

ESERCIZIO 15

(a) $v_1 = \begin{pmatrix} 5 \\ -12 \end{pmatrix} \qquad v_2 = \begin{pmatrix} \sqrt{\pi} \\ -1789 \end{pmatrix}$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$$

$$\tilde{w}_2 = v_2 - \langle v_2 | w_1 \rangle w_1 = \begin{pmatrix} \sqrt{\pi} \\ -1789 \end{pmatrix} - \left(\frac{5}{13}\sqrt{\pi} + \frac{12 \cdot 1789}{13} \right) \begin{pmatrix} 5/13 \\ -12/13 \end{pmatrix} =$$

$$= \begin{pmatrix} 144/169 \sqrt{\pi} - 60/169 \cdot 1789 \\ -1789 + \frac{60}{169}\sqrt{\pi} + \frac{144}{169} \cdot 1789 \end{pmatrix} = \frac{1}{169} \begin{pmatrix} 144\sqrt{\pi} - 60 \cdot 1789 \\ -25 \cdot 1789 + 60\sqrt{\pi} \end{pmatrix}$$

$$w_2 = \frac{\tilde{w}_2}{\|\tilde{w}_2\|} = \begin{pmatrix} A \\ B \end{pmatrix} \cdot \frac{1}{\sqrt{A^2 + B^2}}$$

$A = 144\sqrt{\pi} - 60 \cdot 1789$

$B = 60\sqrt{\pi} - 25 \cdot 1789$

(b) $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\tilde{w}_2 = v_2 - \langle v_2 | w_1 \rangle w_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{4}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4/3 \\ 1 \end{pmatrix}$$

$$\|\tilde{w}_2\| = \sqrt{\frac{16}{3} - \frac{32}{3} + 7} = \sqrt{\frac{5}{3}} \qquad w_2 = \sqrt{\frac{3}{5}} \begin{pmatrix} -4/3 \\ 1 \end{pmatrix}$$

(e) $v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ $\quad v_2 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

$$\|v_1\| = \sqrt{4 + 5 + 27 + 8 + (-6)} = \sqrt{38}$$

$$w_1 = \frac{1}{\sqrt{38}} v_1$$

$$\langle w_1 | v_2 \rangle = \frac{1}{\sqrt{38}} \langle v_1 | v_2 \rangle = \frac{1}{\sqrt{38}} \cdot (6 - 10 + 18 - 4 + 4) =$$

$$= \frac{14}{\sqrt{38}}$$

$$\tilde{w}_2 = v_2 - \langle w_1 | v_2 \rangle w_1 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} - \frac{\cancel{14}^{\,7}}{\cancel{38}_{\,19}} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \frac{1}{19} \begin{pmatrix} 43 \\ -45 \\ 17 \end{pmatrix}$$

$$w_2 = \frac{\tilde{w}_2}{\|\tilde{w}_2\|} = \begin{pmatrix} 43 \\ -45 \\ 17 \end{pmatrix} \Big/ \left\| \begin{pmatrix} 43 \\ -45 \\ 17 \end{pmatrix} \right\| =$$

$$= \begin{pmatrix} 43 \\ -45 \\ 17 \end{pmatrix} \Big/ \sqrt{6631}$$

(f) $v_1 = t - 2t^2$ $\quad v_2 = t$

$$\|v_1\| = \sqrt{\int_0^1 t^2(1-2t)^2 dt} = \sqrt{\int_0^1 t^2 - 4t^3 + 4t^4 \, dt} =$$

$$= \sqrt{\left[\frac{t^3}{3} - t^4 + \frac{4}{5}t^5\right]_0^1} = \sqrt{\frac{1}{3} - 1 + \frac{4}{5}} = \sqrt{\frac{2}{15}}$$

$$w_1 = \sqrt{\frac{15}{2}} (t - 2t^2)$$

$$\langle v_1 | v_2 \rangle = \int_0^1 t \cdot (t - 2t^2) \, dt = \int_0^1 t^2 - 2t^3 \, dt = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

$$\tilde{w}_2 = v_2 - \frac{15}{2} \left(-\frac{1}{6}\right) v_1 =$$

$$= t + \frac{5}{4} (t - 2t^2) = \frac{9}{4} t - \frac{5}{2} t^2$$

$$w_2 = \frac{9t - 10t^2}{\|9t - 10t^2\|} = \frac{9t - 10t^2}{\sqrt{27 + 20 - 45}} = \frac{1}{\sqrt{2}} (9t - 10t^2)$$