Es. Alg. Lin 26/11/2014

Foglio 15/11/2014

Es 5

(c) $V = \{x \in \mathbb{R}^3 : 2x_1 + 3x_2 - 5x_3 = 0\}$

$$B = \left\{ \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{v_1}, \underbrace{\begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix}}_{v_2} \right\} \qquad B' = \left\{ \underbrace{\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}}_{w_1}, \underbrace{\begin{pmatrix} 3 \\ -7 \\ -3 \end{pmatrix}}_{w_2} \right\}$$

$w_1 = \frac{1}{2} v_1 + \frac{1}{2} v_2$

$w_2 = -4 v_1 + 1 v_2$.

matrice $B \to B'$: $M = \begin{pmatrix} \frac{1}{2} & -4 \\ \frac{1}{2} & 1 \end{pmatrix}$

$$v_1 = \frac{2}{5} w_1 - \frac{1}{5} w_2$$

matrice

$$B' \to B : N = \begin{pmatrix} 2/5 & 8/5 \\ -1/5 & 1/5 \end{pmatrix}$$

$$v_2 = 8/5 \, w_1 + 1/5 \, w_2$$

$$M \cdot N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(d) $U = \{ p(t) \in \mathbb{R}_{\leq 2}[t] : p(-1) = 0 \}$

$B = \{ 1+t, 1-t^2 \}$ $\quad B' = \{ 1+2t+t^2, 2-t-3t^2 \}$

$p(t) = a_0 + a_1 t + a_2 t^2$ $\quad a_0 - a_1 + a_2 = 0$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\} \qquad \mathcal{B}' = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \right\}$$

e risolvere come (c)

V è isomorfo a $\left\{ \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} \in \mathbb{R}^3 : a_0 - a_1 + a_2 = 0 \right\}$

ES. 6

$$V = \{ x \in \mathbb{R}^3 : 3x_1 - 2x_2 + 7x_3 = 0 \}$$

$$v = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \qquad \mathcal{B} = \left\{ \underset{b_1}{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}, \underset{b_2}{\begin{pmatrix} 1 \\ 12 \\ 3 \end{pmatrix}} \right\} \qquad \mathcal{B}' = \left\{ \underset{b_1'}{\begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}}, \underset{b_2'}{\begin{pmatrix} -1 \\ 9 \\ 3 \end{pmatrix}} \right\}$$

$$v = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = -\frac{5}{2} b_1 + \frac{1}{2} b_2 \qquad [v]_B = \begin{pmatrix} -5/2 \\ 1/2 \end{pmatrix}$$

$$= \frac{4}{7} b_1' - \frac{1}{7} b_2' \qquad [v]_{B'} = \begin{pmatrix} 4/7 \\ -1/7 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = b_1 = -\frac{1}{7} b_1' + \frac{2}{7} b_2'$$

$$b_2 = \frac{3}{7} b_1' + \frac{8}{7} b_2' \qquad [Id]_B^{B'} = \begin{pmatrix} -1/7 & 3/7 \\ 2/7 & 8/7 \end{pmatrix}$$

$$[Id]_B^{B'} [v]_B = \begin{pmatrix} -1/7 & 3/7 \\ 2/7 & 8/7 \end{pmatrix} \begin{pmatrix} -5/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 4/7 \\ -1/7 \end{pmatrix} = [v]_{B'}$$

ES. 7 $\quad f : V \to W$

$$V = \{ x \in \mathbb{R}^3 , \ x_1 + x_2 - 2x_3 = 0 \}$$

$$W = \{ x \in \mathbb{R}^3 , \ x_1 - x_2 + x_3 = 0 \}$$

$$f(x) = \begin{pmatrix} x_2 - x_1 \\ 5x_3 - x_2 \\ 3x_1 + x_3 \end{pmatrix}$$

conoscere $\longrightarrow$ $[Id]^{B}_{B'}$, $[Id]^{\mathcal{C}}_{\mathcal{C}'}$, $[f]^{\mathcal{C}}_{B}$, $[f]^{\mathcal{C}'}_{B'}$

e verificare $\to$ $[f]^{\mathcal{C}}_{B} = \left([Id]^{\mathcal{C}'}_{\mathcal{C}} \right)^{-1} [f]^{\mathcal{C}'}_{B'} \, [Id]^{B'}_{B}$

$$\left([\mathrm{Id}]^{\mathcal{E}'}_{\mathcal{E}}\right)^{-1} = [\mathrm{Id}]^{\mathcal{E}}_{\mathcal{E}'}$$

$$[f]^{\mathcal{E}}_{\mathcal{B}} = [\mathrm{Id}]^{\mathcal{E}}_{\mathcal{E}'}\,[f]^{\mathcal{E}'}_{\mathcal{B}'}\,[\mathrm{Id}]^{\mathcal{B}'}_{\mathcal{B}}$$

$$\mathcal{B} = \{v_1, v_2\} \qquad \mathcal{B}' = \{v_1', v_2'\}$$

$$\mathcal{E} = \{w_1, w_2\} \qquad \mathcal{E}' = \{w_1', w_2'\}$$

$$f(v_1) = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} = -4 \overset{w_1}{\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}} + 8 \overset{w_2}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}$$

$$f(v_2) = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = -3 w_1 + 4 w_2$$

$$[f]^{\mathcal{E}}_{\mathcal{B}} = \begin{pmatrix} -4 & -3 \\ 8 & 4 \end{pmatrix}$$

$$[f]_{B'}^{C'} = \begin{pmatrix} 13 & -1 \\ -2 & 2 \end{pmatrix}$$

$$[Id]_{B}^{B'} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{pmatrix}$$

$$v_1 = \frac{1}{3} v_1' + \frac{1}{3} v_2'$$

$$v_2 = \frac{1}{3} v_1' - \frac{2}{3} v_2'$$

$$[Id]_{C'}^{C} = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$w_1' = -w_1 + 2 w_2$$

$$w_2' = -w_1 + 3_2$$

$$\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 13 & -1 \\ -2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ 8 & 4 \end{pmatrix}$$

$$[Id]_{C'}^{C} \; [f]_{B'}^{C'} \; [Id]_{B}^{B'} = [f]_{B}^{C}$$