Alg. Lin. 10/12/14

(c) $E: 3x - 5y + 2z = 1$ piani in $\mathbb{R}^3$ non //

$F: 4x + 2y - 7z = 8$ $\Rightarrow E \cap F =$ retta

$E + F = \mathbb{R}^3$

$E \cap F: \begin{cases} \_\_\_\_ \\ \_\_\_\_ \end{cases} = \begin{pmatrix} \vdots \end{pmatrix} + \mathrm{Span}\begin{pmatrix} \vdots \end{pmatrix}$

(d) $E: 2x - 5y + 3z = 1$ $F: \begin{cases} x = -2 + t - s \\ y = 1 + t + 5s \\ z = 7 + t + 9s \end{cases}$

piani in $\mathbb{R}^3$ _

$E \cap F$: $2(-2+t-s)-5(1+t+5s)+3(7+t+9s)=1$

$t(2-5+3)+s(-2-25+27)=1+4+5-21$

$0 \cdot t+0 \cdot s=-11$

$\Rightarrow E \cap F=\emptyset \Rightarrow E \,/\!/\, F$

(In generale: $E=v+W$, $F=u+Z$

$E \cap F \rightsquigarrow$ sist. non omogeneo

$W \cap Z \rightsquigarrow$ dato dal sist. omogeneo associato al prec.)

Per noi: $W \cap Z$: $0 = 0 \quad \forall s, t.$

$\Rightarrow$ tutto il piano

$W = Z$: $2x - 3y + 5z = 0$

$E + F = \mathbb{R}^3.$

$= \mathrm{Span}\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}-1\\5\\3\end{pmatrix}$ —

(e) E: $\begin{cases} 3x - 2y + z = -1 \\ 4x - 3y + 5z = 12 \end{cases}$

F: $\begin{cases} x = 3 + 5t \\ y = -1 + 2t \\ z = 7 - 3t \end{cases}$

rette

$E \cap F$: $\begin{cases} 3(3+5t) - 2(-1+2t) + (7-3t) = -1 \\ 4(3+5t) - 3(-1+2t) + 5(7-3t) = 12 \end{cases}$

$$\begin{cases} t(15-4-3) = -1-9-2-7 \\ t(20-6-15) = 72-12-3-35 \end{cases} \qquad \begin{cases} 8t = -19 \\ -t = -38 \end{cases} \qquad \emptyset$$

$W \cap Z : t = 0 \Rightarrow W \cap Z = \{0\}$

rette sghembe $\quad E + F = \mathbb{R}^3$

(f) $E: \begin{cases} 1+2t \\ -1+5t \\ 4-3t \\ -3+t \end{cases} \qquad F: \begin{cases} 2x - y - 4z + w = -12 \\ 3x + 7y - 6z + 5w = 1 \end{cases}$

retta $E = v + W$

piano $F = u + Z$ $\quad$ in $\mathbb{R}^4$

$E\cap F$	$\dot{W}\cap\overset{2}{Z}$	
$\neq\emptyset$	$\{0\}$	incid. in pto $\Rightarrow$ dim $E+F=3$
	W	$E\subset F$ $\Rightarrow$ dim $E+F=2$
$\emptyset$	$\{0\}$	retta E disgiunta e non parallela al piano F $\Rightarrow$ dim$(E+F)=1+1+2=4$
	W	retta $E\parallel$ piano F dim$(E+F)=1+2=3$

$$E\cap F:\begin{cases}2(1+2t)\pm 1(-1+5t)-4(4-3t)+1(-3+t)=-12\\3(1+2t)+2(-1+5t)-6(4-3t)+5(-3+t)=1\end{cases}$$

$$\begin{cases}t(4\pm5+12+1)=-12-2\pm1+16+3\\t(6+10+18+5)=1-3+2+24+15\end{cases}$$

$$\begin{cases} 22t = 6 \\ 39t = 39 \end{cases} \qquad \begin{cases} 14t = 4 \\ 39t = 39 \end{cases}$$

$E \cap F = \emptyset \qquad W \cap Z = \{0\}$

(Se avessi trovato

$$\begin{cases} 14t = 14 \\ 39t = 39 \end{cases}$$

avrei avuto $E \cap F =$ pto
$W \cap Z = \{0\}$) –

(h) E = piano parallelo $= v + W$
F = piano cat $= u + Z$

In generale:

$E\cap F$	$\dim W\cap Z$
$\neq\emptyset$	$0 \longrightarrow$ piani inc. in un pto $\quad E+F=\mathbb{R}^4$
	$1 \longrightarrow$ piani inc. in rette $\quad \dim E+F=3$
	$2 \longrightarrow$ coincidono ; $\dim E+F=2$
$\emptyset$	$0 \longrightarrow \dim E+F = 1+2+2-0=5$ IMPOSSIBILE
	$1 \longrightarrow$ 2 piani disgiunti non par. $\dim E+F=4$
	$2 \longrightarrow$ paralleli ; $\dim E+F=3$

$$E\cap F: \begin{cases} 3(3+2t-s)-2(-1+5t-4s)+4(1-2t+3s)-5(5-3t+4s)=1 \\ -1(3+2t-s)+3(-1+5t-4s)-4(1-2t+3s)+2(5-3t+4s)=3 \end{cases}$$

$$\begin{cases} t(6-10-8+15)+s(-3+8+12-20)=1-9-2-4+25 \\ t(-2+15+8-6)+s(1-12-12+8)=3+3+3+4-10 \end{cases}$$

$$\begin{cases} 3t-3s=11 \\ 15t-15s=3 \end{cases} \qquad \begin{cases} t-s=11/3 \\ t-s=1/5 \end{cases} \quad \text{imposs}$$

$$\Rightarrow E \cap F = \emptyset$$

$W \cap Z$: $t-s=0 \Rightarrow \dim(W \cap Z)=1$.

② Risolvere :

(a) $z^2 + iz + 2 = 0$

(In gen: $az^2 + bz + c = 0$ ha soluz.

$$z_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \Delta = b^2 - 4ac$$

$\sqrt{\Delta}$: esistono sempre.)

oppure: $z^2 + \alpha z + \beta = (z - z_1)(z - z_2)$

$$= z^2 - (z_1 + z_2) + z_1 z_2$$

dunque: $\begin{cases} z_1 + z_2 = -\alpha \\ z_1 \cdot z_2 = \beta \end{cases}$

Per noi: $\begin{cases} z_1 + z_2 = -i \\ z_1 \cdot z_2 = 2 \end{cases}$ $\qquad z_1 = -2i$, $\quad z_2 = i$

$$\left(z_{1,2} = \frac{-i \pm \sqrt{-1-8}}{2} = \frac{-i \pm 3i}{2} = \begin{cases} i \\ -2i \end{cases} \right).$$

(b) $z^2 - 3z + 3 + i = 0$

$\Delta = 9 - 12 - 4i = -3 - 4i$.

Cerchiamo $\sqrt{\Delta}$ come $\pm(a+ib)$: vogliamo

$$(a+ib)^2 = -3-4i \quad \text{cioè} \quad \begin{cases} a^2 - b^2 = -3 \\ \cancel{2i\,ab = -4i} \\ ab = -2 \end{cases}$$

$a = -1 \quad b = 2$

$$z_{1,2} = \frac{3 \pm (1-2i)}{2} = \begin{cases} 2-i \\ 1+i \end{cases}$$

$$\left(z_1 + z_2 = 3 \qquad z_1 \cdot z_2 = 3+i \right)$$

(c) $z^2 + (1-7i)z - 22 + 7i = 0$

$$\Delta = 1 - 14i - 49 + 88 - 28i = 40 - 42i$$

$\sqrt{\Delta} = \pm (a+ib)$

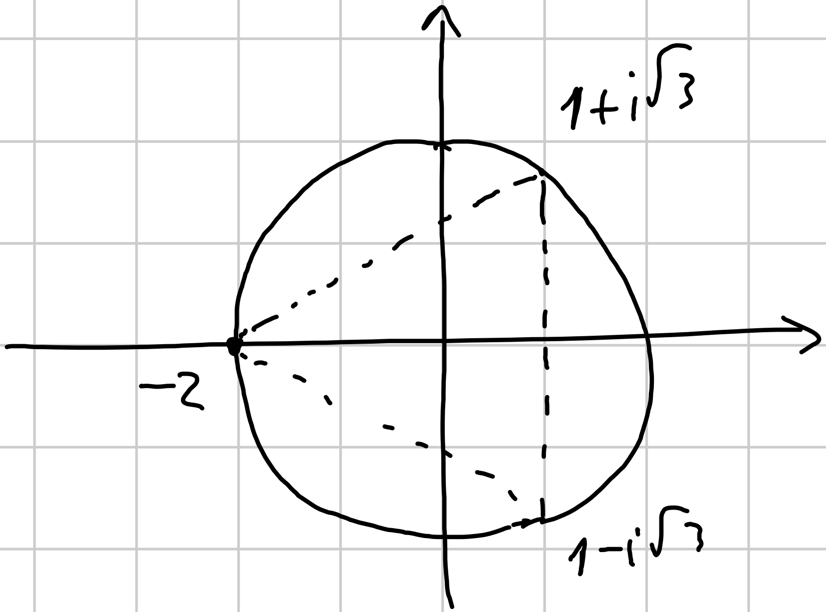
$$\begin{cases} a^2 - b^2 = 40 \\ ab = -21 \end{cases}$$

$a = 7$

$b = -3$

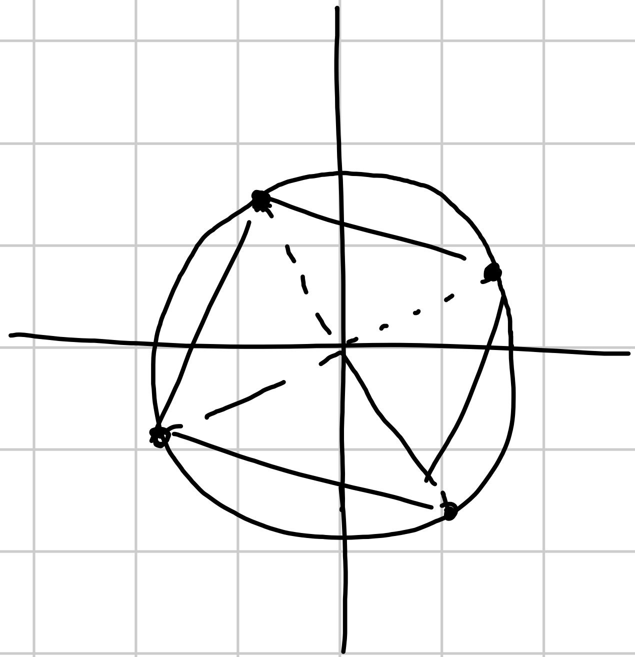
$$z_{1,2} = \frac{-1+7i \pm (7-3i)}{2} = \begin{cases} \ldots \\ \ldots \end{cases}$$

(d) $z^3 = -8$



(e) $z^4 = 4i$ $\quad |4i| = 4 \quad \arg(4i) = \pi/2$

$z_k := \sqrt{2} \cdot e^{i\pi/8 + ik\pi/2} \qquad k = 0, 1, 2, 3$



(f) $3(1+i)z^3 - 4(3+i)z^2 + (13-i)z + 2(i-2) = 0$

ho la soluz $z = 1$.

	$3+3i$	$-12-4i$	$13-i$	$2i-4$
1		$3+3i$	$-9-i$	$4-2i$
	$3+3i$	$-9-i$	$4-2i$	

$(3+3i)z^2 - (9+i)z + 4 - 2i = 0 \qquad \Delta = \dots$

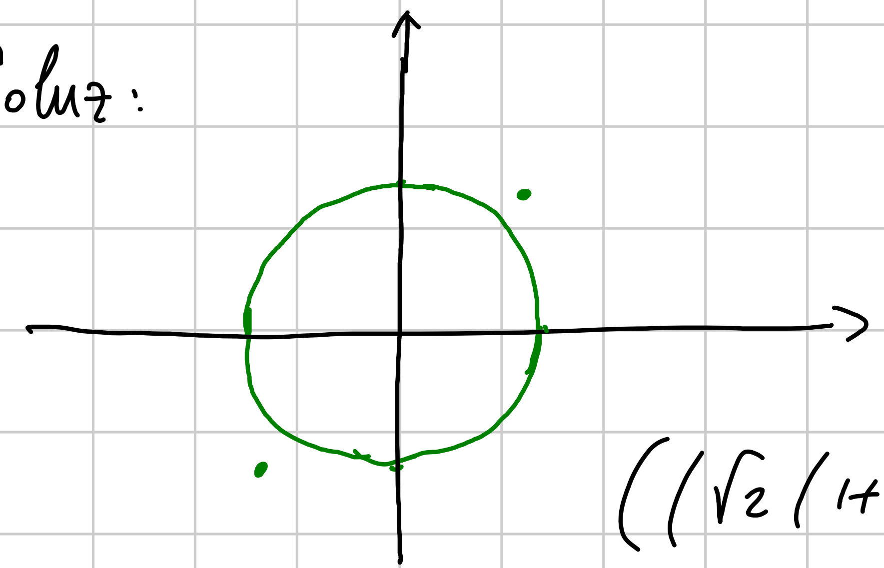
(g) $z^3\bar{z} + 8i = 2z(z + 2i\bar{z})$. $\quad (z\bar{z} = |z|^2)$

equaz. non polinomiale; se scrivo $z = x+iy$ viene sistema di 2 equaz. polinomiali in x e y DIFFICILI

$$z^2|z|^2 + 8i - 2z^2 - 4i|z|^2 = 0$$

$$(|z|^2 - 2)(z^2 - 4i) = 0$$

Soluz:



$$\left((\sqrt{2}(1+i))^2 = 2(1+2i-1) = 4i. \right)$$

(h) $z^2\bar{z} + 4i|z|^2 + i\bar{z} + 4 = z\bar{z}^2 + iz$

$$z|z|^2 + 4i|z|^2 + i\bar{z} + 4 - \bar{z}|z|^2 - iz = 0$$

$$|z|^2(z - \bar{z} + 4i) - i(z - \bar{z} + 4i) = 0$$

$$\underbrace{(|z|^2 - i)}_{\nexists}\underbrace{(2i\,\Im(z) + 4i)}_{\Im(z) = -2} = 0$$

nessuna soluz.

③ Data radice z_1 di $p(z)$ trovare tutte le radici con molt.

(c) $p(z) = z^4 + (1-i)z^3 + (1+3i)z^2 + (9-i)z - 5i$

$z_1 = i$.

	1	$1-i$	$1+3i$	$9-i$	$-5i$
i		i	i	$i-4$	$5i$
	1	1	$1+4i$	$+5$	/
i		i	$i-1$	-5	

	1	$1+i$	$5i$
i		i	$i-2$
	1	$1+2i$	No

$$\Rightarrow p(z) = (z-i)^2 \cdot \left(z^2+(1+i)z+5i\right)$$

$$\Delta = 2i - 20i = -18i$$

$$z_{2,3} = \frac{-(1+i) \pm 3(1-i)}{2}$$

$\mathrm{molt}(i) = 2$

$\mathrm{molt}(z_2) = \mathrm{molt}(z_3) = 1.$

4 $\quad V = \{ z \in \mathbb{C}^4 : i z_1 + (1-i) z_2 - 2 z_3 + (i-1) z_4 = 0 \}$

$v_1 = \begin{pmatrix} 2 \\ i \\ 1+i \\ -1 \end{pmatrix} \in V$: $\qquad \dim V = 4 - 1 = 3$

$\cancel{2i} + \cancel{i+1} \cancel{-2} \cancel{-2i} \cancel{-i+1} = 0$

Completare a base.

$$\begin{pmatrix} 2 \\ i \\ 1+i \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1-i \\ 0 \end{pmatrix} \ \_\_\_$$

⑥

$$\begin{pmatrix} 1 & i & 1-i \\ 2i & 1+3i & -1 \\ 2+i & 1+i & 3i \end{pmatrix}^{-1} = \dots$$

$$\det(A) = 3i - 9 - 2i + 1 + 4i + (1-i)(1+3i)(2+i) + 6i + 1 + i$$

$$= \dots$$

$$\left(A^{-1}\right)_{11} = \frac{(-1)^{1+1}}{\det(A)} \cdot \det\begin{pmatrix} 1+3i & -1 \\ 1+i & 3i \end{pmatrix} = \frac{3i - 9 + 1 + i}{\det(A)} = \frac{-8+4i}{\det(A)}$$

$$\left(A^{-1}\right)_{12} = \frac{(-1)^{1+2}}{\det(A)} \det\begin{pmatrix} i & 1-i \\ 1+i & 3i \end{pmatrix} = \dots$$

(7b) ~~Risolvere~~ al variare di $\alpha \in \mathbb{C}$:

↳ Dire quante soluzioni ha :

$$\begin{pmatrix} 2+i & \alpha+2i \\ \alpha+2-3i & 5+i \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} 1-i \\ \alpha+1 \end{pmatrix}$$

- $\det(\uparrow) = \alpha^2 + (2-i)\alpha + 4i + 6 - 9 - 7i$

$$= \alpha^2 + (2-i)\alpha - 3(1+i)$$

$$\alpha_1 = -3 \qquad \alpha_2 = 1+i$$

Per $\alpha \neq -3, 1+i$ soluz. unica;

$$\alpha = -3 \quad \begin{cases} (2+i)z + (-3+2i)w = 1-i \\ (-1-3i)z + (5+i)w = -2 \end{cases}$$

$$\frac{2+i}{-1-3i} = \frac{(2+i)(-1+3i)}{10} = \frac{-5+5i}{10} = \frac{i-1}{2}$$

$$\left(\frac{-3+2i}{5+i} = \frac{(-3+2i)(5-i)}{26} = \frac{-13+13i}{26} = \frac{i-1}{2} \right)$$

$$\frac{1-i}{-2} = \frac{i-1}{2} \Rightarrow I = \frac{i-1}{2} \cdot II$$

$\Rightarrow$ infinite soluz.

$$\alpha = 1+i \qquad \left(\begin{array}{cc|c} 2+i & 1+3i & 1-i \\ 3-2i & 5+i & 2+i \end{array}\right)$$

$$\underbrace{\begin{matrix} 2+i & 1+3i \\ 3-2i & 5+i \end{matrix}}_{\det = 0}$$

$$\det\begin{pmatrix} 2+i & 1-i \\ 3-2i & 2+i \end{pmatrix} = 4+2i-1-3+3i+2i+2$$
$$= 2+7i \neq 0$$

$\Longrightarrow$ nessuna soluzione _