

Algebra Lineare - Esercizi del 9/10/08

Dire che W sia un sottospazio di V :

(1) $V = \mathbb{R}^n$

(a) $W = \left\{ x \in \mathbb{R}^n : \sum_{j=1}^n (-1)^j x_j = 0 \right\}$

(b) $W = \left\{ x \in \mathbb{R}^n : \sum_{j=1}^n x_j^j = 0 \right\}$

(c) $W = \left\{ x \in \mathbb{R}^n : \prod_{j=1}^n x_j = 0 \right\}$

(d) $W = \left\{ x \in \mathbb{R}^n : \sum_{j=1}^n (j^2 - 3j + 2) x_j = 0 \right\}$

(2) $V = M_{m \times m}(\mathbb{R})$

(a) $W = \left\{ A \in V : a_{11} \cdot a_{mm} = 0 \right\}$

(b) $W = \left\{ A \in V : \sum_{j=1}^k a_{j, m+1-j} = 0 \right\} \quad k = \min(m, m)$

(c) $W = \left\{ A \in V : \sum_{i=1}^m \sum_{j=1}^m \frac{i-j}{i+j} a_{ij} = 0 \right\}$

(d) $W = \left\{ A \in V : a_{1m} = 0, a_{m1} = 0 \right\}$

(e) $W = \left\{ A \in V : |a_{mm}| \leq \sqrt{5} \right\}$

$$(3) \quad V = \mathbb{R}[x]$$

$$(a) \quad W = \{ p(x) \in \mathbb{R}[x] : p(-9) + p'''(7) = 0 \}$$

$$(b) \quad W = \{ p(x) : p(1) \cdot p''(2) = 0 \}$$

$$(c) \quad W = \{ p(x) : \sum_{j=0}^4 (-1)^j x^j \cdot p^{(j)}(x) = 0 \}$$

$$(d) \quad W = \{ p(x) : |p(x)| \leq 1 + e^x \quad \forall x \}$$

$$(e) \quad W = \{ p(x) : \deg(p'(x) - x^2 p'''(x)) \leq 7 \}$$

$$(4) \quad V = \mathcal{F}(\{a, b, c\}, \mathbb{R})$$

$$(a) \quad W = \{ f \in V : f(b) \leq -3f(b) \}$$

$$(b) \quad W = \{ f \in V : f(a) - \sqrt{3}f(b) + 12f(c) = 0 \}$$

$$(c) \quad W = \{ f \in V : f(a) \cdot f(b) + f(a) \cdot f(c) + f(b) \cdot f(c) = 0 \}$$

$$(d) \quad W = \{ f \in V : \exp(-f(a) + 5f(c)) = 1 \}$$

$$(5) \quad V = \mathcal{F}(\mathbb{N}, \mathbb{R})$$

$$(a) \quad W = \{ a \in V : a_{m+3} = m^2 a_m - 2a_{m+2} \quad \forall m \geq 0 \}$$

$$(b) \quad W = \{ a \in V : a_m \cdot a_{m+1} \cdot a_{m+2} = 0 \quad \forall m \geq 0 \}$$

$$(c) \quad W = \{ a \in V : a_m \cdot a_{m+1} = a_m \cdot a_{m+2} = 0 \quad \forall m \geq 0 \}$$