

Algebra Lineare - Esercizi del 4/12/08

Determinare le inverse delle seguenti matrici:

$$(1) \quad \begin{pmatrix} 7 & 2 \\ 1 & -3 \end{pmatrix} \quad \begin{pmatrix} 4 & 3 \\ -1 & 2 \end{pmatrix} \quad \begin{pmatrix} -5 & -7 \\ 3 & 4 \end{pmatrix}$$

$$(2) \quad \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ -3 & 2 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -2 & 7 \\ 4 & 1 & -10 \\ 7 & 5 & -11 \end{pmatrix} \quad \begin{pmatrix} 4 & 2 & 0 \\ 1 & -1 & -2 \\ -5 & 3 & 1 \end{pmatrix}$$

$$(3) \quad \begin{pmatrix} 1 & 2 & -1 & 3 \\ -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 3 & 1 & -1 & 2 \end{pmatrix}$$

Calcolare il rango delle seguenti matrici:

$$(4) \quad \begin{pmatrix} 2 & 1 & 5 & 5 \\ 1 & 2 & 1 & 7 \\ -3 & -1 & -8 & -6 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 1 & -2 & 0 & 1 & 0 \\ -3 & 6 & -1 & -4 & -3 \end{pmatrix}$$

$$(5) \quad \begin{pmatrix} 2 & 1 & 5 & 4 \\ 1 & 3 & 5 & 7 \\ 0 & -2 & -2 & -4 \\ -1 & 0 & -2 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 2 & -3 & 2 \\ 0 & 1 & -2 & -1 \\ 2 & -1 & 4 & 0 \end{pmatrix}$$

Calcolare la dimensione dei seguenti sottospazi affini e trovarne equazioni parametriche quando sono fornite quelle cartesiane, e viceversa:

$$(6) \quad \left\{ x \in \mathbb{R}^3 : \begin{array}{l} 2x_1 + 3x_2 - 4x_3 = 1 \\ 3x_1 - 2x_2 + 5x_3 = -1 \\ 7x_1 + 6x_2 - 3x_3 = 1 \end{array} \right\}$$

$$(7) \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 5 \\ -5 \end{pmatrix} \right)$$

$$(8) \quad \left\{ x \in \mathbb{R}^3 : 2x_1 - 7x_2 + 5x_3 = 4 \right\}$$

$$(9) \quad \begin{pmatrix} 1 \\ 5 \\ -9 \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 7 \\ -4 \\ 11 \end{pmatrix} \right)$$

$$(10) \quad \left\{ x \in \mathbb{R}^3 : \begin{array}{l} x_1 - 2x_2 + x_3 = 5 \\ 2x_1 + 4x_2 - 3x_3 = -2 \\ 5x_1 + 6x_2 - 5x_3 = 1 \end{array} \right\}$$

$$(11) \quad \left\{ x \in \mathbb{R}^4 : \begin{array}{l} 2x_1 - 3x_2 + 4x_3 + 5x_4 = 1 \\ 3x_1 + 4x_2 - 5x_3 + 7x_4 = -2 \end{array} \right\}$$

$$(12) \quad \begin{pmatrix} 2 \\ 1 \\ -3 \\ 4 \end{pmatrix} + \text{Span} \left(\begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \\ 2 \\ 1 \end{pmatrix} \right)$$

Calcolare :

$$(13) \quad \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 7 \\ 2 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix}$$

$$(14) \quad \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \wedge \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix} \wedge \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$$

Calcolare le dimensioni di $E, F, E \cap F, E + F$:

$$(15) \quad E = \left\{ x \in \mathbb{R}^3 : \begin{array}{l} x_1 - 2x_2 + x_3 = 1 \\ 2x_1 + x_2 + 5x_3 = 4 \end{array} \right\}$$

$$F = \left(\begin{array}{c} 2 \\ -1 \\ 3 \end{array} \right) + \text{Span} \left(\begin{array}{c} 5 \\ 3 \\ 2 \end{array} \right)$$

$$(16) \quad E = \left\{ x \in \mathbb{R}^3 : 5x_1 - 4x_2 + 2x_3 = 9 \right\}$$

$$F = \left\{ x \in \mathbb{R}^3 : \begin{array}{l} 2x_1 + 3x_2 + 4x_3 = 1 \\ 3x_1 - 7x_2 - 2x_3 = 4 \end{array} \right\}$$

$$(17) \quad E = \left(\begin{array}{c} 5 \\ -3 \\ 4 \end{array} \right) + \text{Span} \left(\left(\begin{array}{c} 2 \\ 1 \\ -1 \end{array} \right), \left(\begin{array}{c} 3 \\ -2 \\ 1 \end{array} \right) \right)$$

$$F = \left(\begin{array}{c} 2 \\ 1 \\ 5 \end{array} \right) + \text{Span} \left(\left(\begin{array}{c} 4 \\ 3 \\ 2 \end{array} \right) \right)$$

$$(18) \quad E = \left\{ x \in \mathbb{R}^3 : 2x_1 - 3x_2 + 4x_3 = 1 \right\}$$

$$F = \left(\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \right) + \text{Span} \left(\left(\begin{array}{c} 5 \\ 1 \\ -3 \end{array} \right), \left(\begin{array}{c} 2 \\ 2 \\ -1 \end{array} \right) \right)$$

$$(19) \quad E = \left\{ x \in \mathbb{R}^4 : \begin{array}{l} 2x_1 + x_2 + x_3 - x_4 = 1 \\ x_1 - 2x_2 - x_3 = 7 \end{array} \right\}$$

$$F = \left(\begin{array}{c} 2 \\ 1 \\ -1 \\ 0 \end{array} \right) + \text{Span} \left(\left(\begin{array}{c} 3 \\ 1 \\ -2 \\ -1 \end{array} \right), \left(\begin{array}{c} 4 \\ 3 \\ -5 \\ 0 \end{array} \right) \right)$$