

# Geometria e Algebra 1 - Esercizi del 6/12/07

(1) Risolvi in  $\mathbb{C}$ :

(a)  $z^2 - (2+i)z + 3 + 11 \cdot i = 0$

(b)  $2z^3 + z^2 - 2z - 1 - i \cdot (z^2 + 6z - 7) = 0$

(c)  $3z^4 - z^3 + 9z^2 - 4z - 12 - i(z^3 + z^2 + 16z + 4) = 0$

(d)  $z^4 \bar{z} - 2z^3 + |z|^2 = z$

(e)  $z^5 \bar{z} + z(z^3 + \bar{z}) = -1$

(2) Verifica le formule di Grassmann per le seguenti coppie di sottospazi complessi:

(a)  $\{z \in \mathbb{C}^3 : z_1 + iz_2 = (1+i)z_3\}$   
 $\text{Span} \left( \begin{pmatrix} z \\ -i \\ 1 \end{pmatrix}, \begin{pmatrix} 1-i \\ 1 \\ i \end{pmatrix} \right)$

(b)  $\left\{ z \in \mathbb{C}^4 : \begin{aligned} z_1 - iz_2 + 2z_3 - z_4 &= 0 \\ (1+i)z_1 - 2z_2 + iz_3 - 2iz_4 &= 0 \end{aligned} \right\}$

$$\text{Span} \left( \begin{pmatrix} 1 \\ -i \\ 1+i \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1-3i \\ 1-3i \\ 5 \end{pmatrix} \right)$$

(3) Calcolo

$$\det \begin{pmatrix} i & 2-i \\ 7+i & 2+3i \end{pmatrix}$$

$$\det \begin{pmatrix} -1 & i & 2+i \\ 2i & -3 & 1-i \\ -i & 1+2i & 2+i \end{pmatrix}$$

(4) Risolvi

$$\begin{cases} 2iz_1 + (2+3i)z_2 = 4 \\ (1-i)z_1 + (3+i)z_2 = 1-i \end{cases} \quad \begin{cases} (2-i)z_1 + 2iz_2 - 4z_3 = 5 \\ (1+3i)z_1 + z_2 + (1-i)z_3 = 1+i \end{cases}$$

(5) Trova equazioni cartesiane per

$$\left\{ \begin{pmatrix} z-i \\ 7+2i \end{pmatrix} + z \begin{pmatrix} -1+5i \\ z+3i \end{pmatrix} : z \in \mathbb{C} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ -i \\ 2 \end{pmatrix} + z \begin{pmatrix} 1+i \\ 2i \\ 3-4i \end{pmatrix} : z \in \mathbb{C} \right\}$$

$$\left\{ \begin{pmatrix} 1-i \\ 1+i \\ 2i \end{pmatrix} + z \begin{pmatrix} i \\ 2 \\ 1+i \end{pmatrix} + w \begin{pmatrix} -1 \\ 2-i \\ 3+2i \end{pmatrix} : z, w \in \mathbb{C} \right\}$$

(6) Verifica le formule delle dimensioni per

$$f: \mathbb{C}^3 \rightarrow \mathbb{C}^2$$
$$f(z) = \begin{pmatrix} iz_1 - 2z_2 + 4z_3 \\ (2-i)z_2 - iz_3 \end{pmatrix}$$

$$f: \mathbb{C}_{\leq 3}[z] \rightarrow \mathbb{C}_{\leq 2}[z]$$

$$f(p(z)) = p'''(z) + p(i) \cdot z^2 + i p'(-1)$$