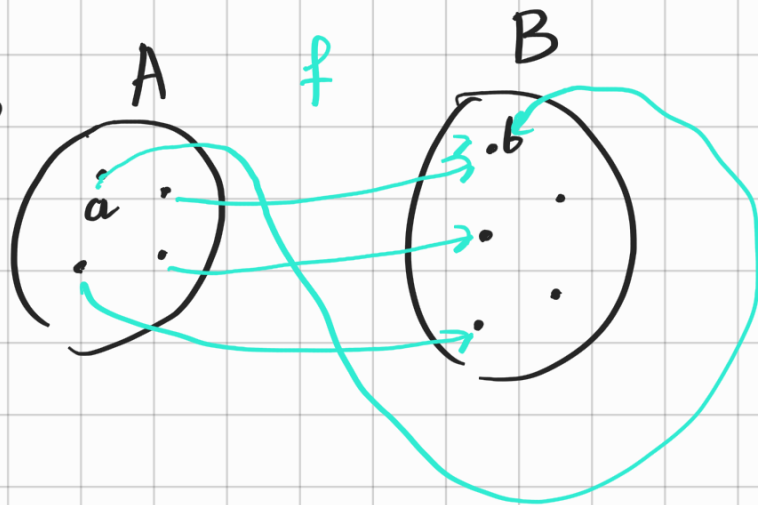
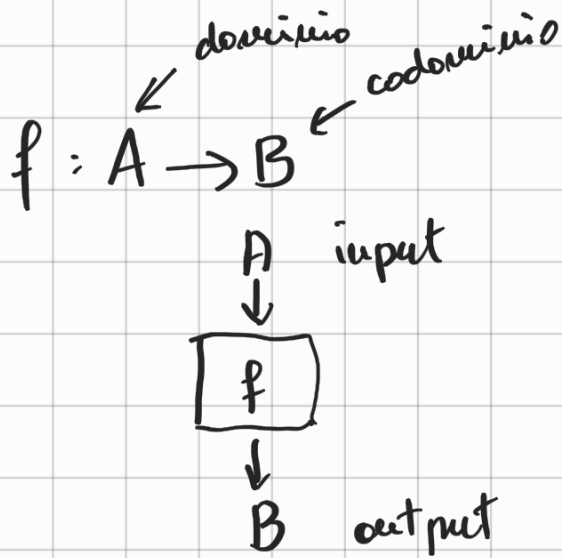


# ANALISI MATEMATICA B

## LEZIONE 6 - 1.10.2021

### FUNZIONI



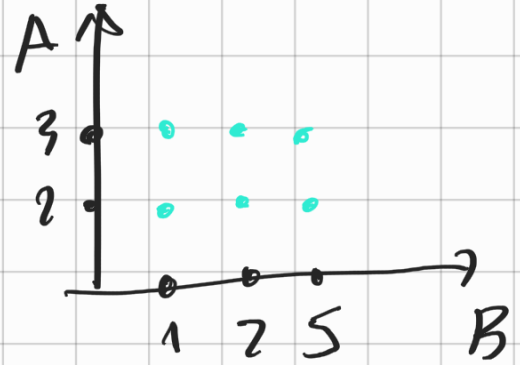
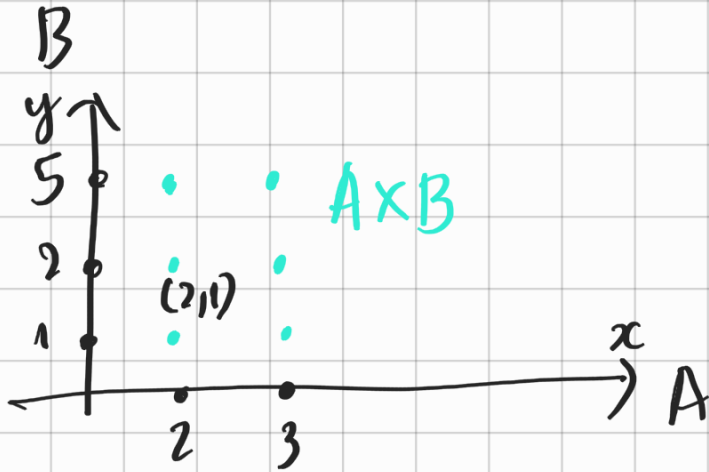
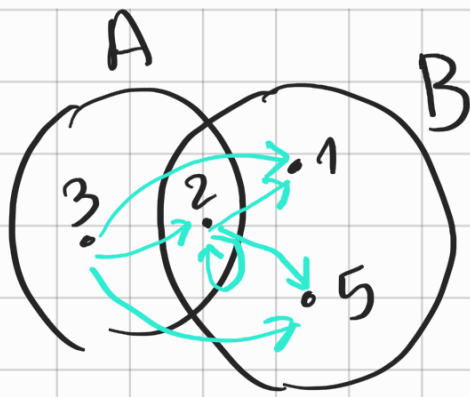
$$a \mapsto b \quad \bar{e} \quad (a, b)$$

prodotto cartesiano

$$A \times B = \{ (a, b) : a \in A, b \in B \}$$

$\uparrow \quad \uparrow$   
 A                      B  
Es.  $\{2, 3\} \times \{1, 2, 5\} = \{ (2, 1), (2, 2), (2, 5), (3, 1), (3, 2), (3, 5) \}$   
 $(\#(A \times B) = (\#A) \cdot (\#B))$

Oss:  $(2, 1) \neq (1, 2)$



ES

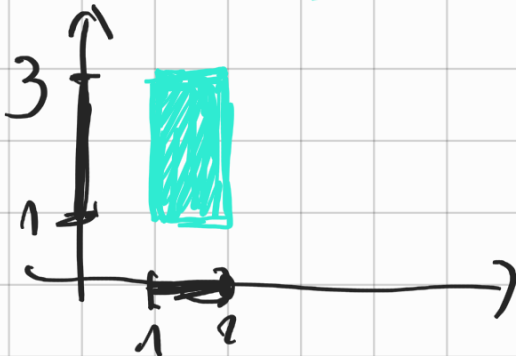
$$A = \{2, 3\}$$

$$A \times \emptyset = \{\} = \emptyset$$

$$A \times \{\emptyset\} = \{(2, \emptyset), (3, \emptyset)\}$$

$\uparrow$ 
 $\uparrow$

$A \times B$



$$A = [1, 2] = \{x \in \mathbb{R} : 1 \leq x \leq 2\}$$

$$B = [1, 3] = \{x \in \mathbb{R} : 1 \leq x \leq 3\}$$

# RELAZIONE

Se  $R$  è un sottoinsieme di  $A \times B$   
diremo che  $R$  è una "relazione" tra  $A$  e  $B$ .

$$\left[ \begin{array}{l} \text{ES } A = \{2,3\} \quad B = \{1,2,5\} \\ \leq = \{ (2,5), (3,5) \} \end{array} \right]$$

Invece di scrivere  $(a,b) \in R$  si può scrivere

$$\boxed{a R b}$$

$$\text{ES } (2,5) \in \leq$$

$$\boxed{2 < 5}$$

$$\left\{ \{ \emptyset, \emptyset \} \right\}, \left\{ \{ \emptyset, \emptyset \}, \dots \right\} \in$$
$$\left\{ \{ \dots \} \right\}$$

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Le relazioni possono avere proprietà particolari:

- simmetrica  $x R y \Leftrightarrow y R x$   
- riflessiva  $x R x$

- transitiva:  $x R y \wedge y R z \Rightarrow x R z$

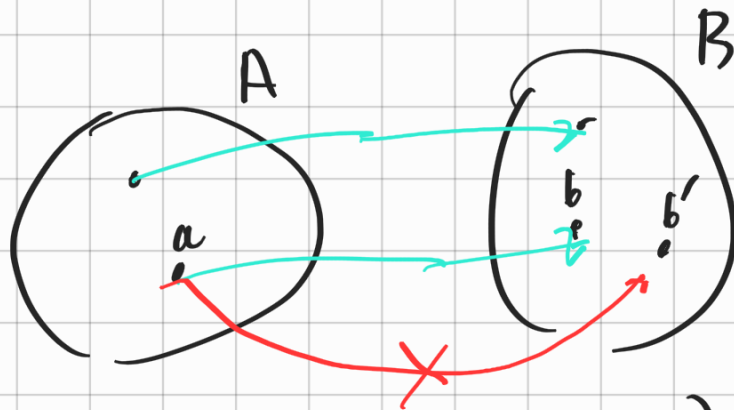
- ...

Diciamo che  $f$  è una funzione da  $A$  a  $B$   
e scriviamo

$$f: A \rightarrow B$$

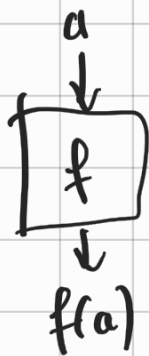
se  $f$  è una relazione da  $A$  in  $B$  tale che:

esistenza (i) definita su tutto  $A$ :  $\forall a \in A \exists b \in B : a \neq b$   
unicità (ii) univoca:  $(a, b) \in f \wedge (a, b') \in f \Rightarrow b = b'$   $\parallel (a \xrightarrow{f} b)$   
 $(a \mapsto b) \in f$



Quindi  $\forall a \in A: \exists! b \in B$  t.c.  $(a, b) \in f$   $a \xrightarrow{f} b$   
tale  $b$  si denota  $f(a)$

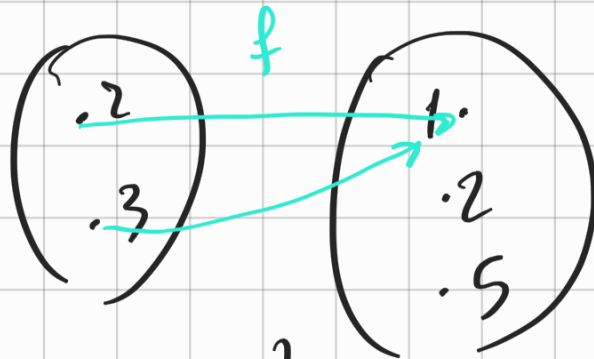
$b = f(a)$  significa  $(a, b) \in f$ .



$$x^{-1} = \left( \frac{1}{x} \right)$$

$$M^{-1}$$

Es 1  $A = \{2, 3\}$      $B = \{1, 2, 5\}$



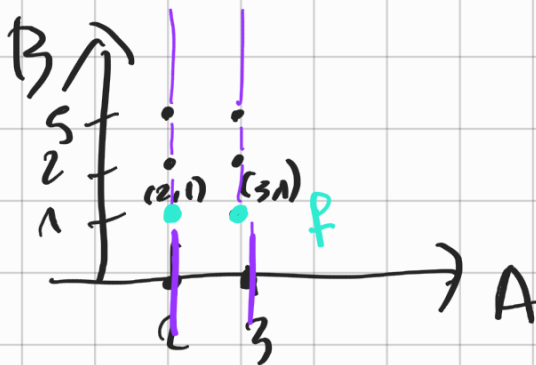
$$f = \{ 2 \mapsto 1, 3 \mapsto 1 \} = \{ (2, 1), (3, 1) \}$$

$f(2) = 1$  ,  $2 \xrightarrow{f} 1$  ,  ~~$(2, 1) \in f$~~

Es 2  $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$      $(f = \{ x \mapsto \frac{1}{x} : x \in \mathbb{R} \})$   
 $x \mapsto \frac{1}{x}$

$(f = \{ (x, \frac{1}{x}) : x \in \mathbb{R} \})$

Es 1

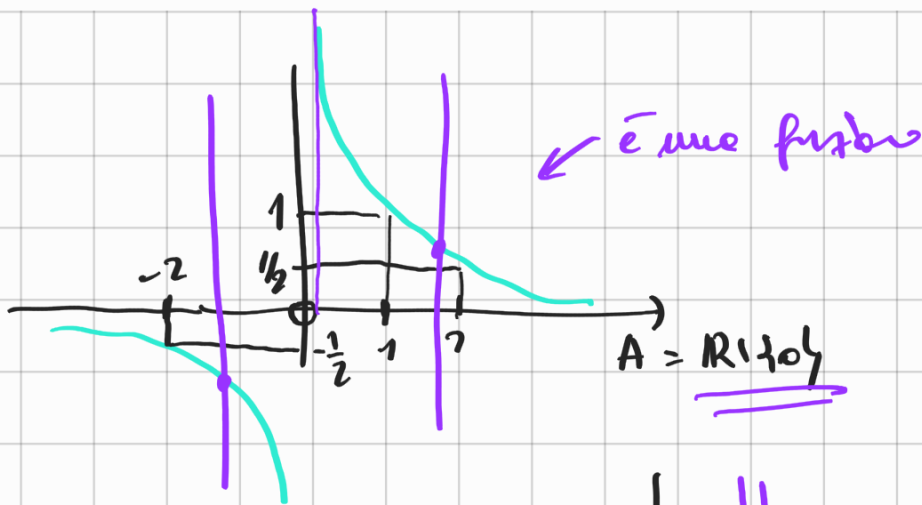


$f: A \rightarrow B$

$\Gamma(f) = \text{grafico di } f = \{ (x, f(x)) : x \in A \}$      $(= f)$  *per noi*

$\Gamma(f) \subseteq A \times B$

Es 2     $f(x) = \frac{1}{x}$      $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$



Non é uma função:  
da  $\mathbb{R} \rightarrow \mathbb{R}$



Non é uma função

