

14 Calcolare



$$\lim_{x \rightarrow 0} \frac{\sin\left(1 - \sqrt[3]{\cos(x^2)}\right)}{\operatorname{arctg}^4 x}$$

$$\cos(x^2) = \cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\cos x^2 = 1 - \frac{x^4}{2} + o(x^4)$$

$$(1+x)^{\frac{1}{3}} = 1 + \frac{x}{3} + o(x)$$

$$\begin{aligned} \sqrt[3]{\cos(x^2)} &= \left(1 - \frac{x^4}{2} + o(x^4)\right)^{\frac{1}{3}} = 1 + \frac{\left(-\frac{x^4}{2} + o(x^4)\right)}{3} + o(x^4) \\ &= 1 - \frac{x^4}{6} + o(x^4) \end{aligned}$$

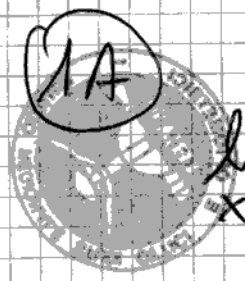
$$\sin\left(1 - \sqrt[3]{\cos(x^2)}\right) = \sin\left(\frac{x^4}{6} + o(x^4)\right) = \frac{x^4}{6} + o(x^4)$$

$$\operatorname{arctg} x = x + o(x)$$

$$(\operatorname{arctg} x)^4 = (x + o(x))^4 = x^4 + o(x^4)$$

$$\frac{\sin\left(1 - \sqrt[3]{\cos(x^2)}\right)}{\operatorname{arctg}^4 x} = \frac{\frac{x^4}{6} + o(x^4)}{x^4 + o(x^4)} = \frac{\frac{1}{6} + \frac{o(x^4)}{x^4}}{1 + \frac{o(x^4)}{x^4}} \rightarrow \frac{1}{6}$$

1A



$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos(\sin^2 x)}}{\arctan^4 x}$$

$$\sin^2 x = (x + o(x))^2 = x^2 + o(x^2)$$

~~$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$~~

$$\cos(\sin^2 x) = 1 - \frac{(x^2 + o(x^2))^2}{2} + o(x^4)$$

$$= 1 - \frac{x^4}{2} + o(x^4)$$

~~$$\sqrt{1+x} = 1 + \frac{1}{2}x + o(x)$$~~

$$\sqrt{\cos(\sin^2 x)} = \sqrt{1 - \frac{x^4}{2} + o(x^4)} = 1 - \frac{x^4}{4} + o(x^4)$$

$$\arctan^4 x = x^4 + o(x^4) \quad (\text{vedi } \boxed{1C})$$

$$\frac{1 - \sqrt{\cos(\sin^2 x)}}{\arctan^4 x} = \frac{\frac{x^4}{4} + o(x^4)}{x^4 + o(x^4)} = \frac{\frac{1}{4} + \frac{o(x^4)}{x^4}}{1 + \frac{o(x^4)}{x^4}} \rightarrow \frac{1}{4}$$

1D

$$\lim_{x \rightarrow 0} \frac{\sin^2(1 - \sqrt[3]{\cos x})}{\arctan^4 x}$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\sqrt[3]{1+x} = 1 + \frac{x}{3} + o(x)$$

$$\sqrt[3]{\cos x} = \left(1 - \frac{x^2}{2} + o(x^2)\right)^{\frac{1}{3}} = 1 - \frac{x^2}{6} + o(x^2)$$

$$1 - \sqrt[3]{\cos x} = \frac{x^2}{6} + o(x^2)$$

$$\sin x = x + o(x)$$

$$\sin\left(1 - \sqrt[3]{\cos x}\right) = \frac{x^2}{6} + o(x^2) + o\left(\frac{x^2}{6} + o(x^2)\right)$$

$$= \frac{x^2}{6} + o(x^2)$$

$$\sin^2\left(1 - \sqrt[3]{\cos x}\right) = \left(\frac{x^2}{6} + o(x^2)\right)^2 = \frac{x^4}{36} + o(x^4)$$

$$\arctan^4 x = x^4 + o(x^4)$$

(Verd! 1D)

$$\frac{\sin^2(1 - \sqrt[3]{\cos x})}{\arctan^4 x} = \frac{\frac{x^4}{36} + o(x^4)}{x^4 + o(x^4)}$$

$$= \frac{\frac{1}{36} + \frac{o(x^4)}{x^4}}{1 + \frac{o(x^4)}{x^4}} \rightarrow \frac{1}{36}$$

0

1B

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt[3]{\cos(\sin(x^2))}}{\arctan^4 x}$$

$$\sin x = x + o(x)$$

$$\sin(x^2) = x^2 + o(x^2)$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\begin{aligned} \cos(\sin(x^2)) &= 1 - \frac{(x^2 + o(x^2))^2}{2} + o((x^2 + o(x^2))^2) \\ &= 1 - \frac{x^4}{2} + o(x^4) + o(x^4) \end{aligned}$$

$$\sqrt[3]{1+x} = 1 + \frac{1}{3}x + o(x)$$

$$\sqrt[3]{\cos(\sin(x^2))} = \sqrt[3]{1 - \frac{x^4}{2} + o(x^4)} = 1 - \frac{x^4}{6} + o(x^4)$$

$$\arctan^4 x = x^4 + o(x^4) \quad (\text{VEDI (1C)})$$

$$\frac{1 - \sqrt[3]{\cos(\sin(x^2))}}{\arctan^4 x} = \frac{1 - (1 - \frac{x^4}{6} + o(x^4))}{x^4 + o(x^4)} = \frac{\frac{x^4}{6} + o(x^4)}{x^4 + o(x^4)} \rightarrow \frac{1}{6}$$

2A



$$f(x) = \frac{\cos x}{1+x^2} = (\cos x) \cdot (1+x^2)^{-1}$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$$

$$(1+x^2)^{-1} = \cancel{1 + 2x + \frac{2(2-1)}{2} x^2 + \frac{2(2-1)(2-2)}{6} x^3 + \frac{2(2-1)(2-2)(2-3)}{24} x^4 + o(x^4)}$$

$$= 1 - x^2 + x^4 + o(x^4)$$

$$(1+x^2)^{-1} = 1 - x^2 + x^4 + o(x^4)$$

$$f(x) = (\cos x) (1+x^2)^{-1} = \left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right) (1 - x^2 + x^4 + o(x^4))$$

$$= 1 - \left(\frac{1}{2} + 1\right) x^2 + \left(\frac{1}{24} + \frac{1}{2} + 1\right) x^4 + o(x^4)$$

$$= 1 - \frac{3}{2} x^2 + \frac{1+12+24}{24} x^4 + o(x^4)$$

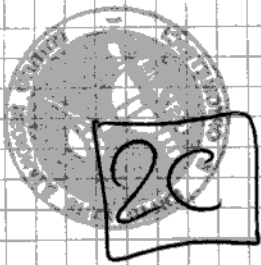
$$= 1 - \frac{3}{2} x^2 + \frac{37}{24} x^4 + o(x^4)$$

$$P_4(x) = 1 - \frac{3}{2} x^2 + \frac{37}{24} x^4 \quad \checkmark$$

2B

è molto simile a 2A

$$\frac{\cos x}{1-x^2} = 1 + \frac{x^2}{2} + \frac{13}{24} x^4$$



(20) e MAUO SIMILE

$$\frac{1-x^2}{\cos x} = 1 - \frac{x^2}{2} + \frac{7x^4}{24} + o(x^4)$$

$$f(x) = \frac{1+x^2}{\cos x} = (1+x^2) (\cos x)^{-1}$$

$$= (1+x^2) \left(1 - \frac{x^2}{2} + \frac{7x^4}{24} + o(x^4) \right)$$

$$(1+x)^k = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + o(x^2)$$

$$\left(1 - \frac{x^2}{2} + \frac{7x^4}{24} + o(x^4) \right)^{-1} =$$

$$= 1 - \left(-\frac{x^2}{2} + \frac{7x^4}{24} + o(x^4) \right) +$$

$$+ \left(-\frac{x^2}{2} + \frac{7x^4}{24} + o(x^4) \right)^2 + o(x^4)$$

$$= 1 + \frac{x^2}{2} - \frac{x^4}{24} + o(x^4)$$

$$+ \frac{x^4}{4} + o(x^4) = 1 + \frac{x^2}{2} + \frac{6-1}{24} x^4 + o(x^4)$$

$$= 1 + \frac{x^2}{2} + \frac{5}{24} x^4 + o(x^4)$$

$$f(x) = (1+x^2) \left(1 + \frac{x^2}{2} + \frac{5}{24} x^4 + o(x^4) \right)$$

$$= \left(1 + \frac{3}{2} x^2 + \left(\frac{1}{2} + \frac{5}{24} \right) x^4 + o(x^4) \right)$$

$$= 1 + \frac{3}{2} x^2 + \frac{17}{24} x^4 + o(x^4) \quad P_4(x) = 1 + \frac{3}{2} x^2 + \frac{17}{24} x^4$$

3A



$$f = \frac{x^2 y - xy}{1 + y^2}$$

$$\frac{\partial f}{\partial x} = \frac{2xy - y}{1 + y^2} = \frac{y(2x - 1)}{1 + y^2}$$

$$\frac{\partial f}{\partial y} = \frac{(x^2 - x)(1 + y^2) - (x^2 y - xy) \cdot 2y}{(1 + y^2)^2}$$

$$= \frac{x[(x - 1)(1 + y^2) - (xy - y)2y]}{(1 + y^2)^2}$$

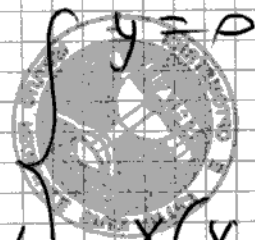
$$= \frac{x[x - 1 + xy^2 - y^2 - 2xy^2 + 2y^2]}{(1 + y^2)^2}$$

$$= \frac{x[x - 1 - xy^2 + y^2]}{(1 + y^2)^2}$$

$$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases} \Rightarrow \begin{cases} y(2x - 1) = 0 \\ x(x - 1 - xy^2 + y^2) = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x(x - 1) = 0 \\ x = \frac{1}{2} \end{cases}$$

$(1 + y^2) \neq 0 \quad \forall y.$

$$\left\{ \frac{1}{2} \left(\frac{1}{2} - 1 - \frac{y^2}{2} + y^2 \right) = 0 \right.$$



$$y=0$$

$$x(x-1)=0$$

$$\begin{cases} y=0 \\ x=0 \end{cases}$$

$$\begin{cases} y=0 \\ x=1 \end{cases}$$

$$x = \frac{1}{2}$$

$$-1 + y^2 = 0$$

$$x = \frac{1}{2}$$

$$y = 1$$

$$x = \frac{1}{2}$$

$$y = -1$$

4 punti critici: $(0,0)$, $(1,0)$, $(\frac{1}{2}, 1)$, $(\frac{1}{2}, -1)$.

$$\frac{\partial f}{\partial x} = \frac{2xy - y}{1 + y^2}$$

$$\frac{\partial f}{\partial y} = \frac{x[x-1 - xy^2 + y^2]}{(1+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{2y}{(1+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{2x(1+y^2) - (2xy - y)2y}{(1+y^2)^2}$$

$$= \frac{2x + 2xy^2 - 4xy^2 + 2y^2}{(1+y^2)^2}$$

$$= \frac{2x - 2xy^2 + 2y^2}{(1+y^2)^2}$$

(3A)

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \frac{2xy - y}{1+y^2} = \frac{2y}{1+y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{(2x-1)(1+y^2) - (2xy-y) \cdot 2y}{(1+y^2)^2}$$

$$= \frac{(2x-1)(1+y^2) - (2xy-y) \cdot 2y}{(1+y^2)^2} \quad \text{---}$$

$$= \frac{(2x-1)(1+y^2) - (2xy-y) \cdot 2y}{(1+y^2)^2} = \frac{2x-1}{1} = 2x-1$$

$$\left(\frac{\partial f}{\partial x} = 0 \right) = \frac{2x-1}{1} = 2x-1$$

$$\left(x = \frac{1}{2} \right) = 0$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \frac{x^2 - x - x^2 y^2 + xy^2}{(1+y^2)^2}$$

$$= \frac{(-2x^2 y + 2xy)(1+y^2)^2 - (x^2 - x - x^2 y^2 + xy^2) \cdot 2(1+y^2) \cdot 2y}{(1+y^2)^4}$$

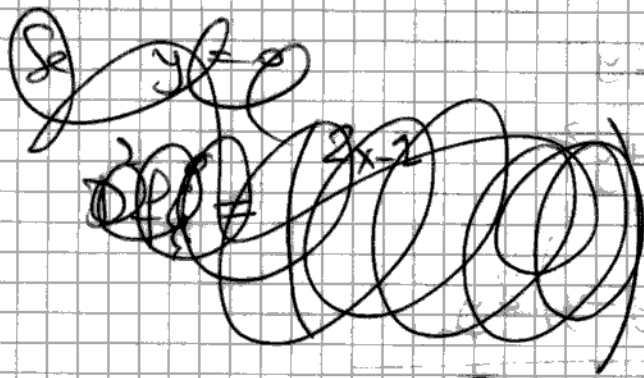
$$= \frac{(-2x^2 y + 2xy)(1+y^2)^2 - (x^2 - x - x^2 y^2 + xy^2) \cdot 2(1+y^2) \cdot 2y}{(1+y^2)^4}$$

$$\left(\frac{\partial f}{\partial y} = 0 \right) = 0$$

$$\left(\frac{\partial f}{\partial y} = 0 \right) = \frac{2xy(1-x) \cdot 2^2 - 0}{2^4}$$

$$= \frac{xy(1-x)}{2} = \left(x = \frac{1}{2} \right) = \frac{y}{8} = \frac{1}{8}$$

$$y = \pm 1$$



$$D^2 f(0,0) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \det = -1 \quad \text{sella}$$

$t_2 = 0$

$$D^2 f(1,0) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \det = -1 \quad \text{sella}$$

$t_2 = 0$

$$D^2 f\left(\frac{1}{2}, 1\right) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{8} \end{pmatrix} \quad \det = \frac{1}{8} \quad \text{min}$$

$t_2 > 0$

$$D^2 f\left(\frac{1}{2}, -1\right) = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{8} \end{pmatrix} \quad \det = \frac{1}{8} \quad \text{max}$$

$t_2 < 0$

3B 3C e 3D

sono molto
SIMILI

4A

$$f = \frac{x^3 y + x^2}{1 + x^2}$$

$$\frac{\partial}{\partial x} \left(x \frac{\partial f}{\partial y} \right) - \frac{\partial}{\partial y} \left(x \frac{\partial f}{\partial x} \right) =$$

$$= \frac{\partial f}{\partial y} + x \frac{\partial^2 f}{\partial x \partial y} - x \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$$

$$= \frac{x^3}{1 + x^2}$$

4B ... $= \frac{\partial f}{\partial y} = \frac{x^2}{1 + x^4}$

4C ... $= \frac{\partial f}{\partial y} = \frac{x^2}{1 + x^2}$

4D ... $= \frac{\partial f}{\partial y} = \frac{x^3}{1 + x^4}$