APPROXIMATE EXTENSION OF PARTIAL ε-CHARACTERS OF ABELIAN GROUPS TO CHARACTERS WITH APPLICATION TO INTEGRAL POINT LATTICES

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Let G be an abelian group, $S \subseteq G$ be a finite set, and $\mathbb{T}$ denote the multiplicative group of complex units with the invariant arc metric $|\arg(a/b)|$.

We will show that for a mapping $f S \to \mathbb{T}$ to be ε-close on S to a character $\varphi G \to \mathbb{T}$ it is enough that f be extendable to a mapping $\bar{f}\left(S \cup \{1\} \cup S^{-1}\right)^n \to \mathbb{T}$, where n is big enough and $\bar{f}$ violates the homomorphy condition at most up to an arbitrary $\delta < \min\left(\varepsilon, \frac{\pi}{2}\right)$. Moreover, n can be chosen uniformly, independently of G and both f and $\bar{f}$, depending just on δ, ε and the number of elements of S.

The proof is non-constructive, using a special case of Gordon's nonstandard version of Pontryagin-van Kampen duality [1], [2] or, alternatively, the ultraproduct construction and the classical Pontryagin-van Kampen duality, hence yielding no estimate on the actual size of n.

As one of the applications we show that, for a vector $u \in \mathbb{R}^q$ to be ε-close to some vector from the dual (polar, reciprocal) lattice $H^\star$ of a full rank integral point lattice $H \leq \mathbb{Z}^q$, it is enough for the scalar product ux to be δ-close (with $\delta < 1/3$) to an integer for all vectors $x \in H$ satisfying $\sum_i |x_i| \leq n$, where n depends on δ, ε and q only.

References

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