RIVER PHENOMENON FOR DIFFERENCE EQUATIONS

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Topic #7: *Nonstandard Methods in Differential Equations.*

We study equations of type

$$D : Y(X+1) = F(X, Y(X))$$

with $X \longrightarrow \infty$ and F nonlinear and C^1 in Y. In analogy with earlier studies on differential equations (among others M.Artigue, V.Gautheron, G. Reeb, F. and M. Diener and the author in the 80's) we study a phenomenon of strong concentrations of trajectories into so-called *rivers*. Rivers tend to occur close to the fix-points $F(X, \widehat{Y}(X)) = \widehat{Y}(X)$ of F. We propose a non-standard mathematical model for this phenomenon, and provide sufficient conditions for the existence of rivers.

Translated into classical terms, the existence theorem yields a new method to determine the asymptotic behavior of solutions of difference equations.

There are some analogies, but also some differences with the rivers of differential equations. Under rather mild conditions the river-phenomenon of differential equations may locally be rescaled to a configuration of exponential behaviour around one solution of nearly constant behaviour. This is not the case for difference equations. We distinguish three types of behaviour: (1) strong contraction, like the behaviour of trajectories in singular perturbations, (2) moderate contraction, with behaviour similar to the set of solutions of difference equations with constant coefficients, and (3) contraction so slow, that on an appropriate scale the solutions are infinitely close to the solutions of the differential equation $y' = \pm y$.

References

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