NON-STANDARD APPROACH TO J.F. COLOMBEAU'S NON-LINEAR THEORY OF GENERALIZED FUNCTIONS AND A DELTA-LIKE SOLUTION OF HOPF'S EQUATION

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[Joint work with Guy Berger.[1]]

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Let $\mathcal{T}$ stand for the usual topology on $\mathbb{R}^d$. J.F. Colombeau's non-linear theory of generalized functions is based on varieties of families of differential commutative rings $\mathcal{G} \stackrel{\text{def}}{=} \{\mathcal{G}(\Omega)\}_{\Omega\in\mathcal{T}}$ such that: 1) Each $\mathcal{G}$ is a **sheaf** of differential rings (consequently, each $f \in \mathcal{G}(\Omega)$ has a **support** which is a closed set of Ω). 2) Each $\mathcal{G}(\Omega)$ is supplied with a chain of sheaf-preserving embeddings $\mathcal{C}^\infty(\Omega) \subset \mathcal{D}'(\Omega) \subset \mathcal{G}(\Omega)$, where $\mathcal{C}^\infty(\Omega)$ is a **differential subring** of $\mathcal{G}(\Omega)$ and the space of L. Schwartz's distributions $\mathcal{D}'(\Omega)$ is a **differential linear subspace** of $\mathcal{G}(\Omega)$. 3) The ring of the scalars $\widetilde{\mathbb{C}}$ of the family $\mathcal{G}$ (defined as the set of the functions in $\mathcal{G}(\mathbb{R}^d)$ with zero gradient) is a non-Archimedean ring with zero devisors containing a copy of the complex numbers $\mathbb{C}$. Colombeau theory has numerous applications to ordinary and partial differential equations, fluid mechanics, elasticity theory, quantum field theory and more recently to general relativity. The main purpose of our **non-standard version of Colombeau' theory** is the improvement of the scalars: in our approach the set of scalars $\widehat{\mathbb{C}}$ is always an algebraically closed non-Archimedean Cantor complete field. This leads to other improvements and simplifications such as reducing the number of quantifiers and possibilities for an axiomatization of the theory. As an application we shall prove the existence of a weak soliton-like solution of Hopf's equation improving a similar result, due to M. Radyna, obtained in the framework of V. Maslov's theory.

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