A MODEL OF QUANTUM FIELD THEORY WITH A FUNDAMENTAL LENGTH

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Topic #8: *Nonstandard Methods in the study of Navier-Stokes equations and in Mathematical Physics.*

The relativistic equation of quantum mechanics called Dirac equation

$$\frac{\hbar}{c}\gamma_\mu \frac{\partial}{\partial x_\mu}\psi(x) + M\psi(x) = 0,\ x_0 = ct, x_1 = x, x_2 = y, x_3 = z$$

contains the constants c (velocity of light) which is the fundamental constant in the relativity theory, and $h = 2\pi\hbar$ (Planck constant) which is the fundamental constant in quantum mechanics. The dimension of c is $[\mathrm{LT}^{-1}]$ and that of h is $[\mathrm{ML}^2\mathrm{T}^{-1}]$. W. Heisenberg thought that the equation must also contain a constant l with dimension [L]. If such l is introduced, the quantities with arbitrary dimensions are expressed by the combination of c, h and l, e.g., $[\mathrm{T}] = [\mathrm{L}]/[\mathrm{LT}^1]$, $[\mathrm{M}] = [\mathrm{ML}^2\mathrm{T}^{-1}]/([\mathrm{LT}^{-1}][\mathrm{L}])$. In 1958, Heisenberg with Pauli introduced the equation

$$\frac{\hbar}{c}\gamma_\mu \frac{\partial}{\partial x_\mu}\psi(x) \pm l^2\gamma_\mu\gamma_5\psi(x)\bar{\psi}(x)\gamma^\mu\gamma_5\psi(x) = 0,$$

which is later called the equation of universe. The constant l has the dimension [L] and is called the fundamental length. But this equation is difficult to solve. So, we consider the following equation having the constant l with the dimension [L]:

$$\begin{cases} \Box\phi(x) = \left(\frac{cm}{\hbar}\right)^2 \phi(x) \\ \left(\frac{\hbar}{c}\gamma_\mu \frac{\partial}{\partial x_\mu} + M\right)\psi(x) = 2i\gamma_\mu l^2 \psi(x)\phi(x)\frac{\partial\phi(x)}{\partial x_\mu} \end{cases} \qquad (1).$$

We construct Schwinger functions to the fields $\phi(x)$ and $\psi(x)$ by means of the path integral on the $*$-finite lattice with an infinitesimal spacing. As a result, the field $\psi(x)$ is not an operator-valued tempered distribution, but an operator-valued tempered ultrahyperfunction. The space of tempered ultrahyperfunctions is the dual space of the space of entire functions rapidly decreasing in any strip $|\mathrm{Im}\, z| \leq M$. Thus, by means of nonstandard analysis, we construct a solution of the equation (1) in the framework of ultrahyperfunction quantum field theory (UHQFT) of E. Brüning and S. Nagamachi "Relativistic quantum field theory with a fundamental length" in J. Math. Phys., 45 (2004) 2199-2231. In this framework, l is the length such that one cannot distinguish events which occur in a distance smaller than l.

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