A COMBINATORIAL INFINITESIMAL REPRESENTATION OF LÉVY PROCESSES AND POSSIBLE APPLICATIONS

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[Topic # 5: *Nonstandard Methods in Measure Theory, Stochastic Analysis, Probability and Statistics.*]

[Joint work with Sergio Albeverio[1]]

Consider a standard $\mathbb{R}^d$-valued Lévy process $x. = (x_t)_{t\in\mathbb{R}_+}$. Also, let $0 < h \approx 0$ and let $\mathbb{L} = \frac{1}{H}{}^*\mathbb{Z}^d \cap [-N,N]^d$ for some $H, N \in {}^*\mathbb{N} \setminus \mathbb{N}$. We will show that there is a right lifting $(X_t)_{t\in h^*\mathbb{N}_0}$ for $x.$ such that $X.$ is, up to drift, the *-independent sum of (1) a multiple of Anderson's random walk associated to the mesh h and (2) an internal jump part with *-independent stationary increments, each consisting of a superposition of stochastic jumps with jump size in $\mathbb{L}$.

The crucial step in the proof of this assertion is a hyperfinite discretisation of the Lévy measure.

For, using a combination of fairly elementary estimates, one can find a sufficient criterion on any internal collection of hyperfinitely many hyperfinite Poisson processes (these processes themselves being, at each time $n{\cdot}h$ for $n \in {}^*\mathbb{N}$, a superposition of n independent, identically distributed stochastic jumps) which ensure that the internal infinitesimal generator associated to the superposition of these processes actually has the same pointwise standard part as the sum of the infinitesimal generators of these hyperfinite Poisson processes.

On the other hand, results by Tom Lindstrøm [*Hyperfinite Lévy processes*, Stochastics and Stochastics Reports **76** (2004), 517 – 548] imply that any Lévy process has a lifting whose jump part's internal infinitesimal generator can be written as a sum of infinitesimal generators of such a collection of hyperfinite Poisson processes which satisfies exactly the aforementioned criterion.

For the sake of an application, note that the thus constructed lifting X – which is a hyperfinite Lévy process in the sense of Lindstrøm's and therfore right-lifts its standard part – is, given $h \approx 0$ and $\mathbb{L}$, unique modulo the order in which the jumps of different sizes and the infinitesimal "jumps" corresponding to Anderson's random walk occur. Hence, modulo permutations of the jumps and the discretised diffusion part, X can be regarded as a binomial hyperfinite Markov chain.

However, binomial Markov chains, when interpreted as stock price models, admit Δ-hedging. Thus, nonstandard analysis allows us to introduce a notion of *Δ-hedging modulo jump permutations* which can be performed in any continuous time Lévy stock price model, in spite of their incompleteness in the classical sense.

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