MATHEMATICS IN A HYPERFINITE WORLD

E. GORDON

Topic #2: *Nonstandard Methods in Algebra, Algebraic Geometry and Topology.*

A new point of view on the correlation between discrete and continuous mathematics became rather popular during a few last decades. According this point of view the continuous mathematics is an approximation of the discrete one, but not vice versa as it is accepted in the classical approach.

The best of all this new point of view was expressed by D. Zeilberger:[1] "Continuous analysis and geometry are just degenerate approximations to the discrete world... While discrete analysis is conceptually simpler ... than continuous analysis, technically it is usually much more difficult. Granted, real geometry and analysis were necessary simplifications to enable humans to make progress in science and mathematics....".

This approach to the correlation between discrete and continuous mathematics can be naturally formalized in the nonstandard analysis, where hyperfinite sets serve as an idealization of very big finite sets. The families of not very big (accessible) elements of very big sets are formalized as σ-subsets of hyperfinite sets and relations of indiscernibility are formalized as π-relations on hyperfinite sets. Continuous objects are obtained under this approach by identifying of indiscernible accessible elements of hyperfinite sets (the construction of nonstandard hull).

This approach to the construction of harmonic analysis on locally compact abelian groups on the base of harmonic analysis on hyperfinite commutative groups was more or less systematically implemented in the author's monograph "Nonstandard methods in commutative harmonic analysis" AMS, Providence RI, 1977.

In this talk we present a review of later results obtained in this direction. In particular, it is proved (L.Yu. Glebsky, E.I. Gordon, C.W. Henson) that locally compact fields cannot be obtained by the construction of nonstandard hull from hyperfinite associative rings. It is shown also (L.Yu. Glebsky, E.I. Gordon, C.J. Rubio) that every locally compact group can be obtained by a construction of nonstandard hull from a hyperfinite left quasigroups (algebras with one binary operation such that every equation of the form $a \cdot x = b$ has a unique solution). This representation yields the existence of the Haar measure. Moreover, a locally compact group is unimodular iff it can be obtained from hyperfinite quasigroups (both left and right simultaneously) by the construction of nonstandard hull. These results show that one of the technical difficulties of discrete analysis mentioned above is caused by the lack of good algebraic properties (e.g. the associativity) in many of finite structures. Some other results are discussed also. All discussed results have a natural reformulation in standard term of approximation of continuous structures by finite ones.

In the joint paper of P. Andreev and the author "A theory of hyperfinite sets" (accepted by APAL) a ε-theory of hyperfinite sets, which does not involve the standardness predicate, is constructed. The construction of nonstandard hull has a natural formalization in this theory. All classical properties of continuous objects obtain by the construction of nonstandard hull can be proved in this theory.

Eastern Illinois University, USA
E-mail address: Y.Gordon@mchsi.com, cfyig@eiu.edu

[1]See e.g. http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/real.html.