

# POLYNOMIAL AND FUNCTIONAL INTERPRETATIONS: CONSTRUCTIVE FOUNDATIONS FOR NON-STANDARD ARITHMETIC

YVON GAUTHIER

Topic #1: *Nonstandard Theories and Models, and Foundations of Non-standard Methods.*

Gödel's Dialectica interpretation (see [3]) has inspired many workers in the field of proof mining of theorems in classical analysis. Kohlenbach, Ferreira and Oliva in particular have made use of the functional interpretation to obtain constructive or feasible versions of important theorems in set theory or functional analysis. Gödel's motivation was intuitionistic in extending the finitist point of view by a functional interpretation over all finite types. It is also the case for the Curry-Howard isomorphism between formulas and types (or sets). The polynomial translation (see [2]) of the functional interpretation is a more radical enterprise and attempts to define an isomorphism between formulas and polynomials for a non-standard arithmetic, the Fermat-Kronecker theory of forms (or homogeneous polynomials) with the constructive principle of infinite descent substituting for Peano's induction postulate for standard arithmetic. Neither bounded arithmetic (see [1]) for subsystems of Peano's arithmetic, nor predicative arithmetic (see [4]) for (open) Robinsons arithmetic seem to capture the full constructive content of classical arithmetic. We present the outline of the polynomial translation in terms of a finite arithmetic with infinite descent.

## References

- (1) Buss S.R. (1986), *Bounded Arithmetic*, Bibliopolis, Napoli
- (2) Gauthier, Y. (1998), *A polynomial translation of Gödel's functional interpretation*, Bulletin of the Section of Logic, University of Lodz, vol. 27, no. 3, 130-137.
- (3) Gödel, K. (1958), *Über eine noch nicht benützte Erweiterung des finiten Standpunktes*, Dialectica 12, 230-237.
- (4) Nelson, E. *Predicative Arithmetic*, Princeton University Press, Princeton, N. J.

DEPARTMENT OF PHILOSOPHY, UNIVERSITY OF MONTREAL, CANADA  
E-mail address: yvon.gauthier@umontreal.ca