HOW TO MEASURE THE SIZE OF SETS

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The naïve idea of "size" for collections seems to obey both to *Aristotle's Principle*: "the whole is greater than its parts" and to *Cantor's Principle*: "1-to-1 correspondences preserve size". Notoriously, Aristotle's and Cantor's principles are incompatible for infinite collections. Cantor's theory of cardinalities weakens the former principle to "the part is not greater than the whole", but the outcoming cardinal arithmetic is very unusual. It does not allow for inverse operations, and so there is no direct way of introducing *infinitesimal* numbers. (Sizes are added by means of disjoint unions and multiplied by means of disjoint unions of equinumerous collections.)

We investigate what happens if we maintain Aristotle's principle, weakening instead Cantor's principle: "equinumerous collections are in 1-1 correspondence". In this way we obtain a very nice arithmetic: in fact, our "numerosities" may be taken to be *nonstandard integers* and the set of ordinals is naturally embedded in them.

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