Some tips for writing mathematics

Davide Lombardo

This document contains a few suggestions for writing your first paper (or thesis!). Full disclosure: (small) parts of what follows are adapted from other sources, notably *Writing in Mathematics* by Annalisa Crannell and J.S. Milne's page `https://www.jmilne.org/math/words.html`.

Keep in mind that I am by no means an expert in writing mathematics (is anyone?), nor a native English speaker. However, I do believe – or at least I hope – that the general suggestions that follow can help beginners get started on the long road towards writing good mathematics. I also recommend that everyone watch `https://www.youtube.com/watch?v=ECQyFzzBHlo&t=2285s`: in this great lecture Jean-Pierre Serre, one of the masters of modern mathematics, explains (in what is basically a mathematician's version of stand-up comedy...) how to write mathematics *badly*.

1. Mathematical symbols should not be used as verbs! For example, instead of writing

   The numbers $x, y \in \mathbb{R}^+$, so their logarithms are well defined

   one should definitely prefer

   The numbers x, y **belong to / lie in** $\mathbb{R}^+$, so their logarithms are well defined

   or

   x and y are positive real numbers, so their logarithms are well defined

2. Quantifiers and symbols for logical connectives should in fact be avoided in the main body of the text altogether! Write "the above holds for all $x \in \mathbb{R}$" rather than "the above holds $\forall x \in \mathbb{R}$", and *definitely* write 'implies' instead of '$\Rightarrow$'. On the other hand, it's perfectly fine to use quantifiers like $\forall$ and $\exists$ in centred formulas.

3. Avoid weasel words. There are some notorious phrases that advertise that you don't really understand the logic you need. Be wary of phrases like "clearly", "obviously", and "the only way this can happen is". When I wrote my bachelor's dissertation, my advisor told me that it would have been easy for him to put me in a tight spot: he only had to look for instances of the word 'clearly' in what I'd written and ask me to explain what formal arguments would be needed to justify each of them. Even

experienced mathematicians fall into the trap of hiding their mistakes with a well-placed 'clearly': I'm not saying you shouldn't use the word at all, but please do so with great care. In particular, try to be aware of what you could replace it with, should anyone ask for explanations.

4. 'Then' is supposed to introduce *consequences* of *hypotheses* or of previous arguments. For example, I don't particularly like statements such as:

   Let n be a natural number. Then $\exists a, b, c, d \in \mathbb{Z} : n = a^2 + b^2 + c^2 + d^2$.

   The sentence preceding 'then' is not a *hypothesis*, it's the *definition* of the symbol n. Also, the statement could do with more words and fewer symbols:

   For every natural number n there exist integers a, b, c, d such that $n = a^2 + b^2 + c^2 + d^2$.

   The following would be very different:

   Let n be a real number. If n is a positive integer, then there exist integers a, b, c, d such that $n = a^2 + b^2 + c^2 + d^2$.

   Here we have an actual *hypothesis*: the real number n is not just any random real number, but is in fact an *integer*!

   If you find this strange, think about it in terms of formal logic: it wouldn't make sense to write
   $$n \Rightarrow \exists a, b, c, d \in \mathbb{Z} : n = a^2 + b^2 + c^2 + d^2.$$
   The correct statement is of course
   $$\forall n \in \mathbb{N}\, \exists a, b, c, d \in \mathbb{Z} : n = a^2 + b^2 + c^2 + d^2.$$

   Here's another example: instead of writing

   Let $f : E_1 \to E_2$ be a morphism of elliptic curves with $f(0) = 0$. Then f is a group homomorphism.

   (which in any case is not too bad), consider writing either

   Every morphism of elliptic curves $f : E_1 \to E_2$ such that $f(0) = 0$ is a group homomorphism.

   or

Let $f : E_1 \to E_2$ be a morphism of elliptic curves. If $f(0) = 0$, then f is a group homomorphism.

5. Don't write one sentence when you should write two! Constructions like

   Let X be an object of type A, it is easy to see that X satisfies [...]

   don't quite make sense: the part starting with 'it is easy to see' is simply a *new* sentence. Don't be afraid of using full stops where they are needed:

   Let X be an object of type A. It is easy to see that X satisfies [...]

   Or choose a different formulation entirely:

   For every object X of type A, it is easy to see that X satisfies [...]

6. Ideally, two different formulas should not appear side by side, but should always be separated by at least one word, so as to make it clear where one formula ends and the other begins. Consider for example the sentence

   since $x, y \in \mathbb{R}$ $\Phi(x, y)$ is also a real number.

   The intended meaning is that a certain quantity $\Phi(x, y)$, which may in principle be a complex number, is in fact real, because the arguments x, y are themselves real numbers. However, this is not immediately clear at first glance, and the typesetting doesn't help. It would be better to write

   as $x, y \in \mathbb{R}$, the quantity $\Phi(x, y)$ is also a real number.

   or (even better)

   since x, y are real numbers, $\Phi(x, y)$ is also a real number / the complex number $\Phi(x, y)$ is in fact real.

   For similar reasons, a sentence should never start with a mathematical formula.

7. There is a commonly adopted (though not universal) convention that names of *numbered* statements should be written with a capital letter, while names of unnumbered statements should be written in lower-case. So, for example one should write

   combining Theorem 3.7 and the previous lemma we obtain...

8. (For native Italian speakers) Beware of false friends! 'Thesis' does *not* mean 'statement, conclusion'. *Never* write 'The thesis follows' (which would be incomprehensible to a native speaker), but rather 'The claim follows', or 'The lemma follows', or 'The conclusion follows', or choose a different formulation altogether ('The proof is finished', 'and we are done'). Do *not* use 'verify' when you mean 'satisfy': *to verify* means *to check*. Quoting from J. S. Milne:

   "Verify" is often misused by mathematicians for "satisfy", especially by those whose native language is French. For example, Roos (2006) writes: "The reason for this is that AB4 is rarely verified for them." He means satisfied. It is possible for a condition to be always satisfied but rarely verified (for example, the commutativity of a certain class of diagrams).* ***verify*** *means* ***to prove the truth of****;* ***satisfy*** *means* ***to fulfill the requirements of****. For example, I believe the function satisfies the Leibniz condition, but I haven't verified [i.e., checked] this.*

   In general, I cannot recommend enough that the reader of the present text set it aside immediately and consult `https://www.jmilne.org/math/words.html` instead. There, among other marvels you'll find out that you *shouldn't use "associate to"* and that *the verbs "permit", "allow", "reduce" normally require objects!*

9. Make it clear where the hypotheses are being used. If a hypothesis appears in a statement but not in its proof, it is very likely that at least one of the following is true:

   (a) the hypothesis is unnecessary: you should then remove it from the statement.

   (b) the proof is incorrect, because it is *not* making use of a necessary hypothesis. This is of course the worst-case scenario, and you should definitely fix the problem!

   (c) the proof is correct, but the hypothesis is being used surreptitiously, without the reader being made aware of it. Please help the poor reader by guiding them step-by-step through your thought process!

10. Miscellanea. 'Analogue' is a noun, while 'analogous' is an adjective (so write either 'Our next result is an analogue of the implicit function theorem in the algebraic setting', or 'The following result is analogous to Lemma 3.4'). *Continuous* and *discontinuous* end in -uous (two u's!). The linux *aspell* package provides the ability to spell-check LaTeX files:

```
aspell --lang=en --mode=tex check file.tex
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