Updating incomplete factorizations for PDEs

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The solution of algebraic linear systems of partial differential equations often requires the solution of sequences of large and sparse linear systems of the form

$$A_{\alpha_j} x = b, \qquad A_{\alpha_j} = A + \alpha_j E_j, \quad j = 0, ..., s, \quad \alpha_j \in \mathbf{C}, \tag{1}$$

and E_j, $j = 0, ..., s$ are banded complex matrices.

These linear systems are often solved by iterative methods. However, without preconditioning these methods can converge very slowly and computing an incomplete factorization for each of the linear systems (1) can be costly and require memory space especially if s is large. On the other hand, reusing the same incomplete factorization strategy (e.g., computed for A) often leads to slow convergence. Clearly, there is a broad range of possibilities within these two extremes. We found a strategy for the update of an existing incomplete factorization preconditioner at a cost much lower than recomputing an incomplete factorization from scratch. Even if the resulting preconditioner can be expected to be less effective than a brand new one in terms of iteration count, the overall cost is, under suitable conditions, considerably reduced.

The numerical solution of sequences of algebraic linear systems from the discretization of the real and complex Helmholtz equation and of the diffusion equation illustrate the performance of the proposed approaches.

Sequences of linear systems as in (1) arise also in the numerical solution of the Helmholtz and of the Schrödinger equation; in large and sparse eigenvalues computation; in algorithms for nonlinear least squares and total least squares problems; in globalized quasi-Newton algorithms, etc.