

Corso di laurea in Scienze biologiche molecolari
Corso di Matematica e statistica B - Lista di esercizi n.1

1. Completare la seguente tabella:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
$\cos x$								
$\sin x$								
$\tan x$								
x	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos x$								
$\sin x$								
$\tan x$								

2. Calcolare $\cos \frac{\pi}{12}$, $\sin \frac{5\pi}{12}$, $\tan \frac{\pi}{8}$, $\cos \frac{5\pi}{8}$, $\sin \frac{7\pi}{8}$, $\tan \frac{11\pi}{12}$.
3. Dalle formule di addizione e sottrazione per seno e coseno, dedurre le seguenti *formule di prostaferesi*:

$$\left\{ \begin{array}{l} \cos \alpha + \cos \beta = 2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} \\ \cos \alpha - \cos \beta = -2 \sin \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2} \\ \sin \alpha + \sin \beta = 2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} \\ \sin \alpha - \sin \beta = 2 \cos \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2} . \end{array} \right. \quad \text{per ogni } \alpha, \beta \in \mathbb{R}.$$

4. Provare le disuguaglianze

$$\sin x < x < \tan x \quad \text{per } 0 < |x| < \frac{\pi}{2}.$$

5. Risolvere le equazioni:

- (i) $3 \sin x - \sqrt{3} \cos x = 0$, (ii) $2 \sin^2 x - \sin x - 1 = 0$,
(iii) $\sin x + \frac{6}{\pi} \cos x = 1$, (iv) $\sin^4 x - 4 \sin^2 x \cos^2 x + 3 \cos^4 x = 0$,
(v) $\sin x + 3|\sin x| = 2$, (vi) $\cos^4 x - 4 \sin^2 x \cos^2 x + 3 \sin^4 x = 0$.

6. Risolvere le disequazioni:

- (i) $\sin x < \frac{1}{2}$, (ii) $4 \sin x \tan x - \frac{3}{\cos x} > 0$,
(iii) $\cos x > -\frac{1}{\sqrt{2}}$, (iv) $\begin{cases} \tan x > \sqrt{3} \\ \sin x > \frac{1}{2} \end{cases}$,
(v) $\frac{\sqrt{2} \sin x - 1}{\sqrt{2} \sin x + 1} > 0$, (vi) $\sin x + (\sqrt{2} - 1) \cos x > \sqrt{2} - 1$.

7. Risolvere le seguenti equazioni:

- (i) $\sqrt{8^x} = \frac{1}{4}$; (ii) $9^{1/(x-1)} = 3^{1/(3x-1)}$;
(iii) $7^{x^2-5x+9} = 343$; (iv) $\frac{(5^{2-x})^{3+x}}{25^{x-1}} = \frac{(5^{x-2})^{2x-3}}{25^{2x} \cdot 125^3}$;
(v) $\begin{cases} x + y = 4 \\ 3^{xy} = 27 \end{cases}$; (vi) $\begin{cases} x^2 + y^2 = 17 \\ 5^{x+y} = 125 \end{cases}$;
(vii) $8^{\frac{5x^2-3}{x^2+1}} = 2^{\frac{15-3x^2}{3x^2+1}}$; (viii) $81^{2x-1} + 2 \cdot 9^{4x} + 711 = 81^{2x+1} + \frac{1}{9}$.

8. Risolvere le seguenti equazioni e disequazioni:

- (i) $7^{x+1} + 7^{x-1} = 5^x$; (ii) $\sqrt{4^x} - 15\sqrt[4]{4^x} = 16$;
(iii) $3^{x+1} \geq 5^{1-x}$; (iv) $\frac{1}{2} < |2^x - 1| < 2$;
(v) $\log_3 x - \log_{1/3} x > 2$; (vi) $\log_{1/2}(2x+3) \leq 3$;
(vii) $|\log_{10} |x|| = 100$; (viii) $(3 - 2^x)(5^{x/2} - 2) > 0$;
(ix) $\log_3 (\log_4(x^2 - 5)) < 0$; (x) $\log_2 |x| \leq 3 - \log_4 |x|$;
(xi) $\log_4 x^2 - \log_8 \sqrt{x} = \frac{5}{3}$; (xii) $\log_{2x} x < \frac{1}{2}$;
(xiii) $\begin{cases} y^x = 10^4 \\ y^{1/x} = 10 \end{cases}$; (xiv) $\begin{cases} xy = 1/2 \\ x^{\log_2 y} = \frac{1}{4} \end{cases}$.