ANA 1 | gi 29/11/18

Esercizi su $\mathbb{C}$

- $\begin{cases} \bar{z} w = 1 \\ z^2 + w^2 = -2 \end{cases}$

- $z|z|^2 + |z|\bar{z}^2 + \bar{z} z^2 = i$

- $|z^2+i|^2 + |z^2-i|^2 = 8z^2 - 6$

- $z^3 = \arg z + \frac{\pi}{3}$ $\quad (\arg z \in [0, 2\pi[)$

- $z^2 = -i|z| - \sqrt{2}$

- $z + |z| = 3i + a, \quad a \in \mathbb{R}$

- $\begin{cases} |z^2+1| = 1 \\ \mathrm{Re}\, z = \frac{1}{2}|z|^2 \end{cases}$

- $|z| z^2 \bar{z}^3 i = 1$

- $z - 2|z| + 3\bar{z} = 4i$

Esercizi su successioni

- $\lim_{n\to\infty} \dfrac{(n+1)^{n+1}-n^n}{(n+1)!-n!}$

- $\lim_{n\to\infty} \dfrac{n^2+3n+1}{n^\alpha\left[\sqrt{2n+1}-\sqrt{n}\right]}$, $\alpha\in\mathbb{R}$.

- $\lim_{n\to\infty} \dfrac{2^{(2^{n+1})}-2^{(2^n+1)}+1}{2^{(2^n+1)}\left(2^{2^n}-1\right)}$

- $\lim_{n\to\infty} \left(\dfrac{n^2+1}{n^2-2}\right)^n$

- $\lim_{n\to\infty} \dfrac{n^{7/4}+\ln(n^5+1)}{n\ln n!+\sqrt{1+n^3}}$

- $\max\lim_{n\to\infty}$ e $\min\lim_{n\to\infty}$ di $(-1)^n\cdot n^{(-1)^n}$

- $\sup_{n\in\mathbb{N}}$, $\inf_{n\in\mathbb{N}}$, $\max\lim_{n\to\infty}$, $\min\lim_{n\to\infty}$ di $(-1)^{\frac{n(n+1)}{2}}\dfrac{n+5}{n+2}$; esibire una sottosuccessione convergente.

- $\lim_{n\to\infty}\left[n^2-n^3\sin\frac{1}{n}\right]$

Esercizi su serie

- $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{(x+2)^{n^2}}$, $x\in\mathbb{R}\setminus\{-2\}$.

- $\displaystyle\sum_{n=0}^{\infty} \left(x-\frac{(-1)^n}{5}\right)^n$, $x\in\mathbb{R}$. Calcolare la somma (quando esiste).

- $\displaystyle\sum_{n=1}^{\infty} (-1)^n \log\left(1+\frac{n+n^2}{n^\alpha}\right)$, $\alpha\in\mathbb{R}$

- $\displaystyle\sum_{n=1}^{\infty} \frac{\lambda^n\left(\frac{3}{2}-\lambda\right)^{n^2}}{n^2}$, $\lambda\in\mathbb{R}$.

- $\displaystyle\sum_{n=1}^{\infty} \cos n\pi \left(\cos^2\frac{1}{\sqrt{n}}-\cos\frac{1}{n}\right)$

- $\displaystyle\sum_{n=0}^{\infty} a_n$, ove $a_n=\begin{cases} 2^{-n}, & n \text{ pari} \\ ne^{-8n}, & n \text{ dispari} \end{cases}$.

- $\displaystyle\sum_{n=0}^{\infty} a_n$, ove $a_n=\begin{cases} x^n, & n \text{ pari} \\ e^{nx^3}, & n \text{ dispari} \end{cases}$, $x\in\mathbb{R}$.

- $\displaystyle\sum_{n=1}^{\infty} \frac{i^n}{n}$.

- $\displaystyle\sum_{n=0}^{\infty} x^n\cos(n+1)y$: Raggio di convergenza e somma $(x,y\in\mathbb{R})$.

- $\sum_{n=0}^{\infty} \left[\sqrt{n+1} - \sqrt{n}\right] (1-x^2)^n$, $\quad x \in \mathbb{R}$.

- Per $\alpha \in \mathbb{R}$, calcolare $\lim_{n\to\infty} (n^2+1)^{1-\alpha} \frac{n^\alpha+1}{n^\alpha}$; per quali α la serie $\sum_{n=1}^{\infty} (n^2+1)^{1-\alpha} \frac{n^\alpha+1}{n^\alpha}$ converge?

- Si calcoli $\ell(a) = \lim_{n\to\infty} \left(\frac{a^n+3^n}{5^n+2^n}\right)^{\frac{1}{n}}$ $\quad (a \geq 0)$ e si analizzi la serie

$$\sum_{n=1}^{\infty} \left[\left(\frac{a^n+3^n}{5^n+2^n}\right)^{\frac{1}{n}} - \ell(a)\right].$$

- Per $0 < \alpha < 1$ poniamo

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} \quad \forall n \geq 1, \quad \binom{\alpha}{0} = 1$$

(coefficiente binomiale generalizzato). Provare che:

$\lim_{n\to\infty} \binom{\alpha}{n} = 0$, analizzare $\sum_{n=0}^{\infty} \binom{\alpha}{n}$, e trovare il raggio di convergenza di $\sum_{n=0}^{\infty} \binom{\alpha}{n} x^n$.

- $\sum_{n=0}^{\infty} \frac{1}{\binom{3n}{2n}} z^n$.

- $\sum_{n=1}^{\infty} \frac{(1+i)^{2n}}{2^n (n+4)}$.